CP Violation

CERN Academic Training

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1) Introduction

P, C and T operations:

Basic definition

P = parity transformation $\boldsymbol{x} \rightarrow -\boldsymbol{x}$

C = charge conjugation $e \rightarrow -e$

T = time reversal $t \rightarrow -t$

Extension

P: $\boldsymbol{p} \rightarrow -\boldsymbol{p}$, $\boldsymbol{L} \rightarrow \boldsymbol{L}$, $\boldsymbol{s} \rightarrow \boldsymbol{s}$, $\boldsymbol{j} \rightarrow -\boldsymbol{j}$, $\boldsymbol{E} \rightarrow -\boldsymbol{E}$, $\boldsymbol{B} \rightarrow \boldsymbol{B}$

C: $\boldsymbol{p} \rightarrow \boldsymbol{p}$, $\boldsymbol{L} \rightarrow \boldsymbol{L}$, $\boldsymbol{s} \rightarrow \boldsymbol{s}$, $\boldsymbol{j} \rightarrow -\boldsymbol{j}$, $\boldsymbol{E} \rightarrow -\boldsymbol{E}$, $\boldsymbol{B} \rightarrow -\boldsymbol{B}$

T: $\boldsymbol{p} \rightarrow -\boldsymbol{p}$, $\boldsymbol{L} \rightarrow -\boldsymbol{L}$, $\boldsymbol{s} \rightarrow -\boldsymbol{s}$, $\boldsymbol{j} \rightarrow -\boldsymbol{j}$, $\boldsymbol{E} \rightarrow \boldsymbol{E}$, $\boldsymbol{B} \rightarrow -\boldsymbol{B}$

Further extension

C: particle $\rightarrow$ anti-particle

NB

electric dipole interaction $\propto \boldsymbol{s} \bullet \boldsymbol{E}$ P: $\rightarrow -\boldsymbol{s} \bullet \boldsymbol{E}$ T: $\rightarrow -\boldsymbol{s} \bullet \boldsymbol{E}$

2) Kaon System

$$\left|\mathrm{K}^{+}\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \left|\mathrm{K}^{0}\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad s = +1$$

and

two iso-doublet states

$$\left|\overline{\mathrm{K}}^{0}\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \left|\mathrm{K}^{-}\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad s = -1$$

Under strong interactions, s is conserved.

$\rightarrow$ kaons cannot decay strongly.

The s-quark can change its flavour with weak interactions

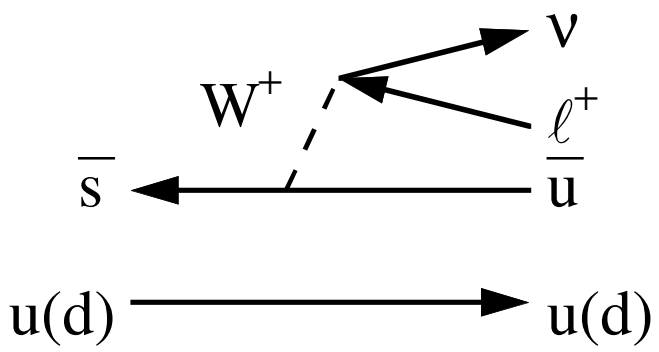
$\Delta s = \pm 1$ processes: decays of kaons

Semileptonic decays: $K \to \pi \ell \nu$, $2\pi \ell \nu$ etc.
Leptonic decays: $K \to \ell \nu$, $\ell \ell$ etc.
Hadronic decays: $K \to 2\pi$, 3π etc.

decay amplitude

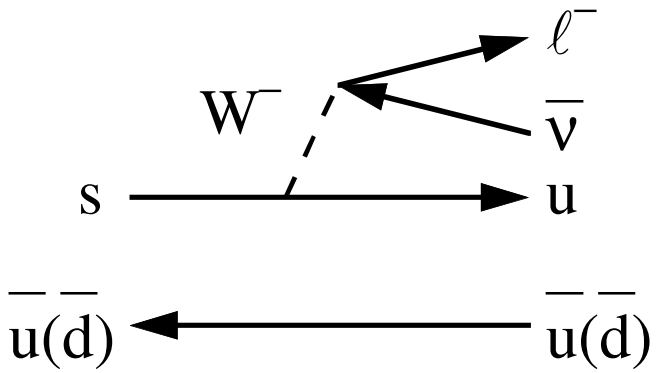
$$A_f = {}_{out}\langle f \mid H_w \mid K \rangle$$

semileptonic decays: with tree diagrams



$K^+(K^0) \to \ell^+ \nu$ + hadrons

$s = +1$	0	$\Delta s = -1$
$Q = +1\ (0)$	$0\ (-1)$	$\Delta Q = -1$



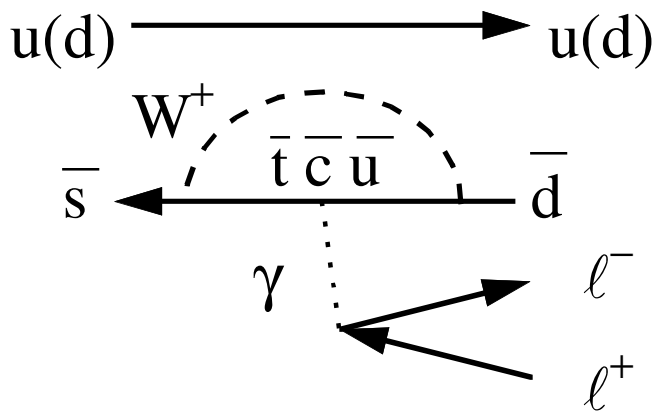
$K^-(\overline{K^0}) \to \ell^- \bar{\nu}$ + hadrons

$s = -1$	0	$\Delta s = +1$
$Q = -1\ (0)$	$0\ (+1)$	$\Delta Q = +1$

$$\Delta Q = \Delta s \text{ rule}$$

another type of semileptonic decays: with penguin diagrams

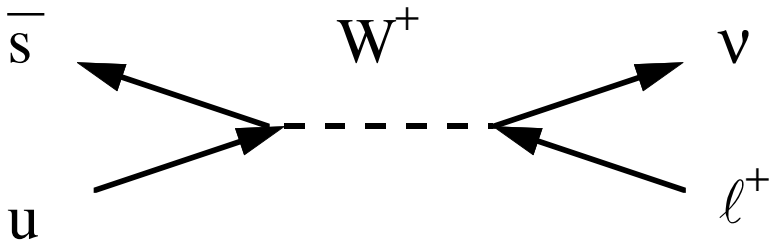
for example



$$K^+(K^0) \to \ell^+ \ell^- + \text{hadrons}$$
$$+ \text{c.c.}$$

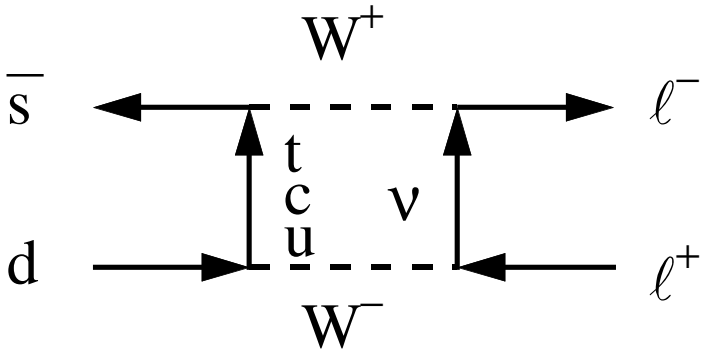
leptonic decays:

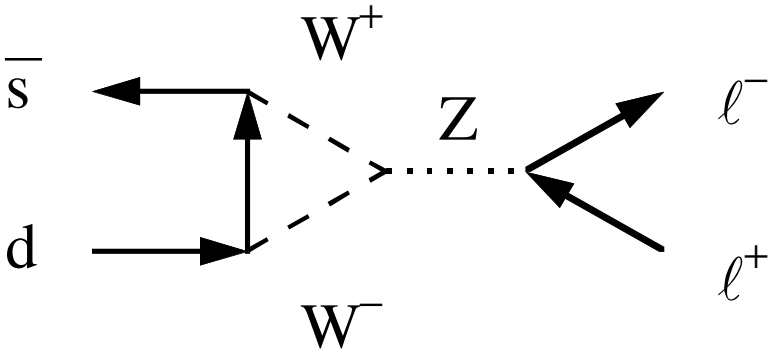
1st order electroweak interactions



W-annihilation diagram

2nd order electroweak interactions

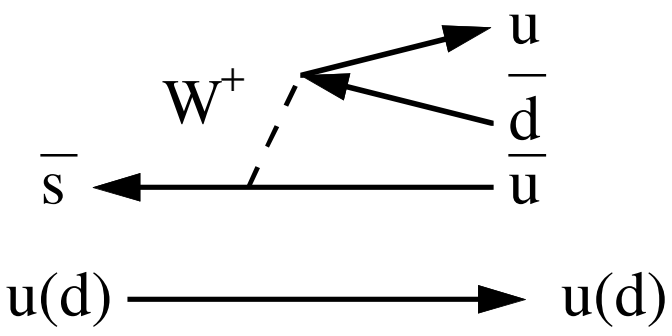


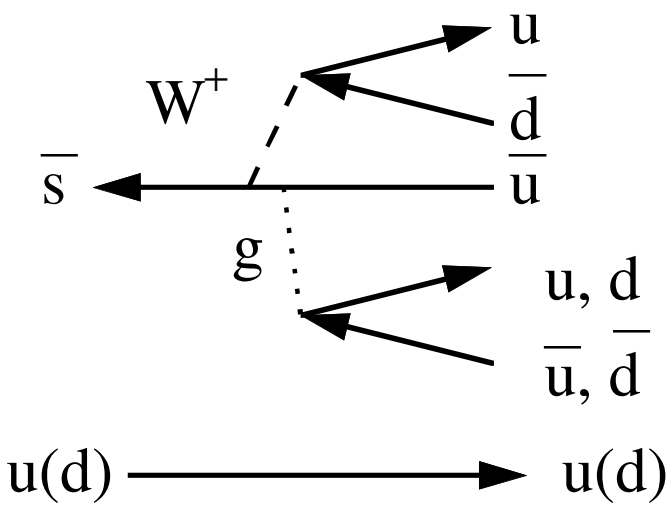


box diagrams

hadronic decays:

Tree diagrams



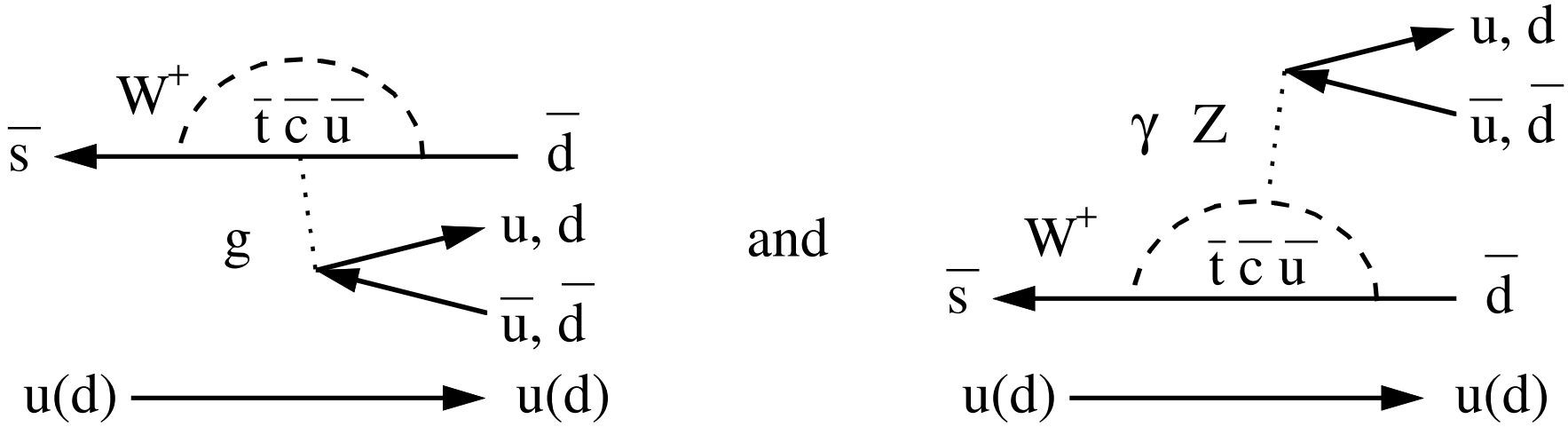


$K^+(K^0) \to \pi^+\pi^0\,(\pi^+\pi^-,\ \pi^0\pi^0)$

$K^+(K^0) \to \pi^+\pi^0\pi^0,\ \pi^+\pi^+\pi^-$
$(\pi^+\pi^-\pi^0,\ \pi^0\pi^0\pi^0)$

+ c.c.

Penguin diagrams



$$K^+(K^0) \to \pi^+\pi^0\,(\pi^+\pi^-,\ \pi^0\pi^0)$$

+ more gluons

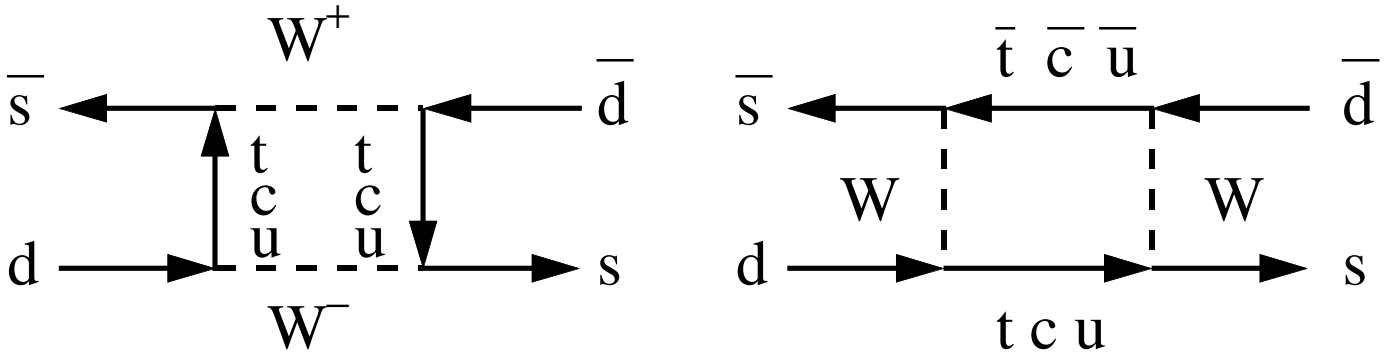
(NB: isospin breaking etc., discussed later)

$\Delta s = \pm 2$ processes: K^0-$\overline{K}^0$ oscillations through **common** states

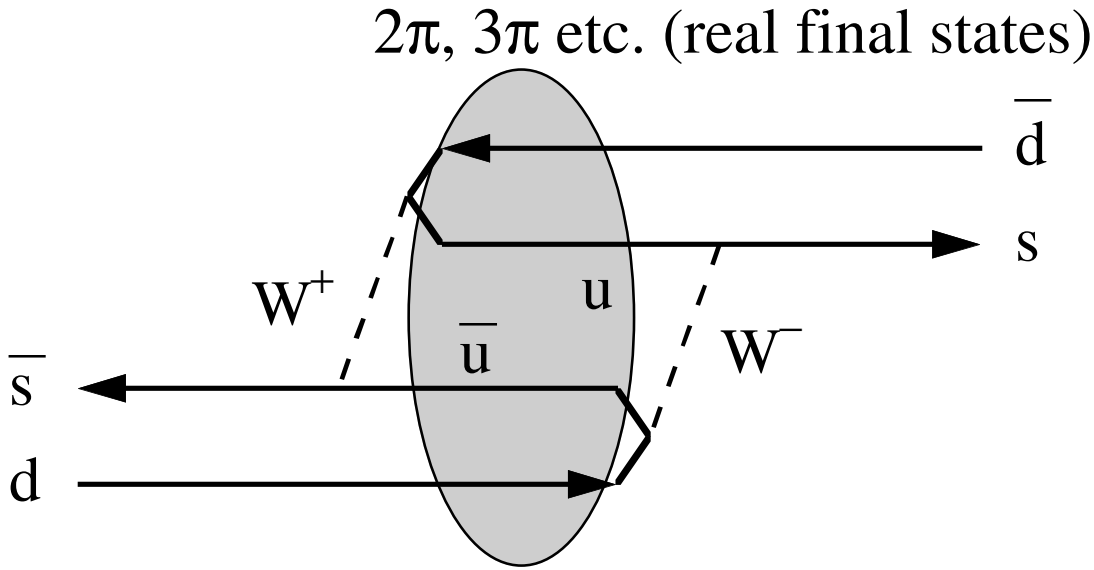
2nd order electroweak interactions

box diagrams

dispersive



absorptive



3) Particle Antiparticle System

K^0 and $\overline{K}^0$ **at rest** (non relativistic system)

$$P\left|K^0\right\rangle = -\left|K^0\right\rangle,\ P\left|\overline{K}^0\right\rangle = -\left|\overline{K}^0\right\rangle$$

$$CP\left|K^0\right\rangle = e^{i\phi}\left|\overline{K}^0\right\rangle,\ T\left|K^0\right\rangle = e^{i\theta}\left|K^0\right\rangle,$$

$$CP\left|\overline{K}^0\right\rangle = e^{i\overline{\phi}}\left|K^0\right\rangle,\ T\left|\overline{K}^0\right\rangle = e^{i\overline{\theta}}\left|\overline{K}^0\right\rangle$$

ϕ and φ are arbitrary phases

$(CPT)^2|K^0> = |K^0>$

(kaons are bosons)

$$\overline{\theta} - \theta = \phi - \overline{\phi} \equiv 2\varphi$$

4) Basic Equations

H_0: strong interaction Hamiltonian

$$H_0\left|\mathrm{K}^0\right\rangle = m_{\mathrm{K}}\left|\mathrm{K}^0\right\rangle,\ H_0\left|\overline{\mathrm{K}}^0\right\rangle = m_{\overline{\mathrm{K}}}\left|\overline{\mathrm{K}}^0\right\rangle \qquad m\text{: rest mass}$$

If strong interactions conserve CPT (which is most likely)

$$\left[CPT, H_0\right] = 0$$

$$\begin{aligned} m_{\mathrm{K}} &= \left\langle \mathrm{K}^0 \left| H_0 \right| \mathrm{K}^0 \right\rangle \\ &= \left\langle \mathrm{K}^0 \left| (CPT)^{-1}(CPT)H_0(CPT)^{-1}(CPT) \right| \mathrm{K}^0 \right\rangle \\ &= \left\langle \mathrm{K}^0 \left| (CPT)^{-1} H_0(CPT) \right| \mathrm{K}^0 \right\rangle \\ &= \left\langle \overline{\mathrm{K}}^0 \left| H_0 \right| \overline{\mathrm{K}}^0 \right\rangle \\ &= m_{\overline{\mathrm{K}}} \equiv m_0 \end{aligned}$$

$t = 0 \quad \rightarrow \quad \mathrm{K}^0$

$\quad\quad\quad

switching on weak interactions

$$|\psi(t)\rangle = a(t)\left|\mathrm{K}^0\right\rangle + b(t)\left|\overline{\mathrm{K}}^0\right\rangle + \sum_f c_f(t)|f\rangle$$

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = \left(H_0 + H_{\mathrm{W}}\right)|\psi(t)\rangle \qquad (\ H^\dagger = H : \text{unitarity}\ )$$

f: all possible states related the kaon system through weak interactions

H_{W}: weak interaction Hamiltonian

$$\boxed{H_{\mathrm{W}} << H_0}$$

interaction (Dirac) representation

$$i\frac{\partial}{\partial t}|\psi(t)\rangle_{\mathrm{D}} = H_{\mathrm{W}}(t)|\psi(t)\rangle_{\mathrm{D}}$$

$$|\psi(t)\rangle_{\mathrm{D}} = e^{iH_0 t}|\psi(t)\rangle,\ H_{\mathrm{W}}(t) = e^{iH_0 t}H_{\mathrm{W}}e^{-iH_0 t}$$

$$i\frac{\partial}{\partial t}a(t)_D = \left\langle \mathrm{K}^0 \middle| H_{\mathrm{W}} \middle| \mathrm{K}^0 \right\rangle a(t)_D + \left\langle \mathrm{K}^0 \middle| H_{\mathrm{W}} \middle| \overline{\mathrm{K}}^0 \right\rangle b(t)_D$$

$$+\sum_f e^{i\left(m_0 - E_f\right)t} \left\langle \mathrm{K}^0 \middle| H_{\mathrm{W}} \middle| f \right\rangle c_f(t)_D$$

$$i\frac{\partial}{\partial t}b(t)_D = \left\langle \overline{\mathrm{K}}^0 \middle| H_{\mathrm{W}} \middle| \mathrm{K}^0 \right\rangle a(t)_D + \left\langle \overline{\mathrm{K}}^0 \middle| H_{\mathrm{W}} \middle| \overline{\mathrm{K}}^0 \right\rangle b(t)_D$$

$$+\sum_f e^{i\left(m_0 - E_f\right)t} \left\langle \overline{\mathrm{K}}^0 \middle| H_{\mathrm{W}} \middle| f \right\rangle c_f(t)_D$$

$$i\frac{\partial}{\partial t}c_f(t)_D = e^{i\left(E_f - m_0\right)t} \left\langle f \middle| H_{\mathrm{W}} \middle| \mathrm{K}^0 \right\rangle a(t)_D + e^{i\left(E_f - m_0\right)t}$$

$$\left\langle f \middle| H_{\mathrm{W}} \middle| \overline{\mathrm{K}}^0 \right\rangle b(t)_D + \sum_{f'} e^{i\left(E_f - E_{f'}\right)t} \left\langle f \middle| H_{\mathrm{W}} \middle| f' \right\rangle c_{f'}(t)_D$$

1) Wigner-Weisskopf approximation:

neglecting weak interactions among the final states

$$\langle f | H_{\mathrm{W}} | f' \rangle = 0$$

$$i \frac{\partial}{\partial t} c_f(t)_D = e^{i\left(E_f - m_0\right)t} \langle f | H_{\mathrm{W}} | \mathrm{K}^0 \rangle a(t)_D$$

$$+ e^{i\left(E_f - m_0\right)t} \langle f | H_{\mathrm{W}} | \overline{\mathrm{K}}^0 \rangle b(t)_D$$

2) Partial integration

$$c_f(t)_D = -i\left[\langle f | H_{\mathrm{W}} | \mathrm{K}^0 \rangle a(t)_D + \langle f | H_{\mathrm{W}} | \overline{\mathrm{K}}^0 \rangle b(t)_D\right]$$

$$\times \int_0^t e^{i\left(E_f - m_0\right)t'} dt' + \mathrm{O}\left(\geq H_W^2\right)$$

3) Neglecting higher orders in H_{W}

$$c_f(t)_D = \left[\left\langle f \left| H_{\mathrm{W}} \right| \mathrm{K}^0 \right\rangle a(t)_D + \left\langle f \left| H_{\mathrm{W}} \right| \overline{\mathrm{K}}^0 \right\rangle b(t)_D \right]$$

$$\times \lim_{\varepsilon \to +0} \frac{e^{i\left(E_f - m_0\right)t}}{m_0 - E_f + i\varepsilon}$$

4) And using the relation

$$\lim_{\varepsilon \to +0} \frac{1}{x + i\varepsilon} = P\left(\frac{1}{x}\right) - i\pi\delta(x)$$

we obtain

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \Lambda \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \qquad \Lambda_{ij} = M_{ij} - \frac{i}{2}\Gamma_{ij}$$

$$M_{ij} = m_0\delta_{ij} + \langle i|H_{\rm W}|j\rangle + \sum_f P\left(\frac{\langle i|H_{\rm W}|f\rangle\langle f|H_{\rm W}|j\rangle}{m_0 - E_f}\right)$$

sum over all possible states

$$|1\rangle = \left|\mathrm{K}^0\right\rangle$$

$$\Gamma_{ij} = 2\pi\sum_f \langle i|H_{\rm W}|f\rangle\langle f|H_{\rm W}|j\rangle\delta\left(m_0 - E_f\right)$$

$$|2\rangle = \left|\overline{\mathrm{K}}^0\right\rangle$$

sum over real final states

NB: $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$ are Hermitian, since $H_{\rm W}$ is Hermitian
Λ is **not** Hermitian

$$M_{21} = M_{12}^*,\ \Gamma_{21} = \Gamma_{12}^*$$

$$M_{11},\ M_{22},\ \Gamma_{11},\ \Gamma_{22}\ :\ \text{real}$$

5) T, CP and CPT

If T is conserved

$$[T,\ H_{\mathrm{W}}] = 0$$

$$\begin{aligned}
\left\langle \mathrm{K}^0 \middle| H_{\mathrm{W}} \middle| \overline{\mathrm{K}}^0 \right\rangle &= \left\langle \mathrm{K}^0 \middle| \left(T^{-1}T\right) H_{\mathrm{W}} \left(T^{-1}T\right) \middle| \overline{\mathrm{K}}^0 \right\rangle \\
&= \left\{ \left(\left\langle \mathrm{K}^0 \middle| T^{-1} \right) \left(T H_{\mathrm{W}} T^{-1} T \middle| \overline{\mathrm{K}}^0 \right\rangle \right) \right\}^* \\
&= \left\{ \left\langle \mathrm{K}^0 \middle| H_{\mathrm{W}} \middle| \overline{\mathrm{K}}^0 \right\rangle e^{i\left(\overline{\theta}-\theta\right)} \right\}^* \\
&= \left\langle \overline{\mathrm{K}}^0 \middle| H_W^{\dagger} \middle| \mathrm{K}^0 \right\rangle e^{-i\left(\overline{\theta}-\theta\right)} \\
&= \left\langle \overline{\mathrm{K}}^0 \middle| H_{\mathrm{W}} \middle| \mathrm{K}^0 \right\rangle e^{-2i\varphi}
\end{aligned}$$

a) If *T* is conserved

$$\Lambda_{12} = \Lambda_{21} e^{-2i\varphi}$$

if $\Lambda_{11} \neq \Lambda_{22}$

CP and CPT are **BOTH**
not conserved
(no knowledge on T)

b) If *CP* is conserved

$$\Lambda_{12} = \Lambda_{21} e^{-2i\varphi}$$

$$\Lambda_{11} = \Lambda_{22}$$

if $|\Lambda_{12}| \neq |\Lambda_{21}|$

c) If *CPT* is conserved

$$\Lambda_{11} = \Lambda_{22}$$

CP and T are **BOTH**
not conserved
(no knowledge on CPT)

6) Solutions of Basic Equations (assuming CPT)

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \Lambda \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \qquad (\Lambda_{11} = \Lambda_{22} \equiv \Lambda_0)$$

$$\ddot{a}(t) + 2i\Lambda_0 \dot{a}(t) + \left(\Lambda_{12}\Lambda_{21} - \Lambda_0^2\right) a(t) = 0$$

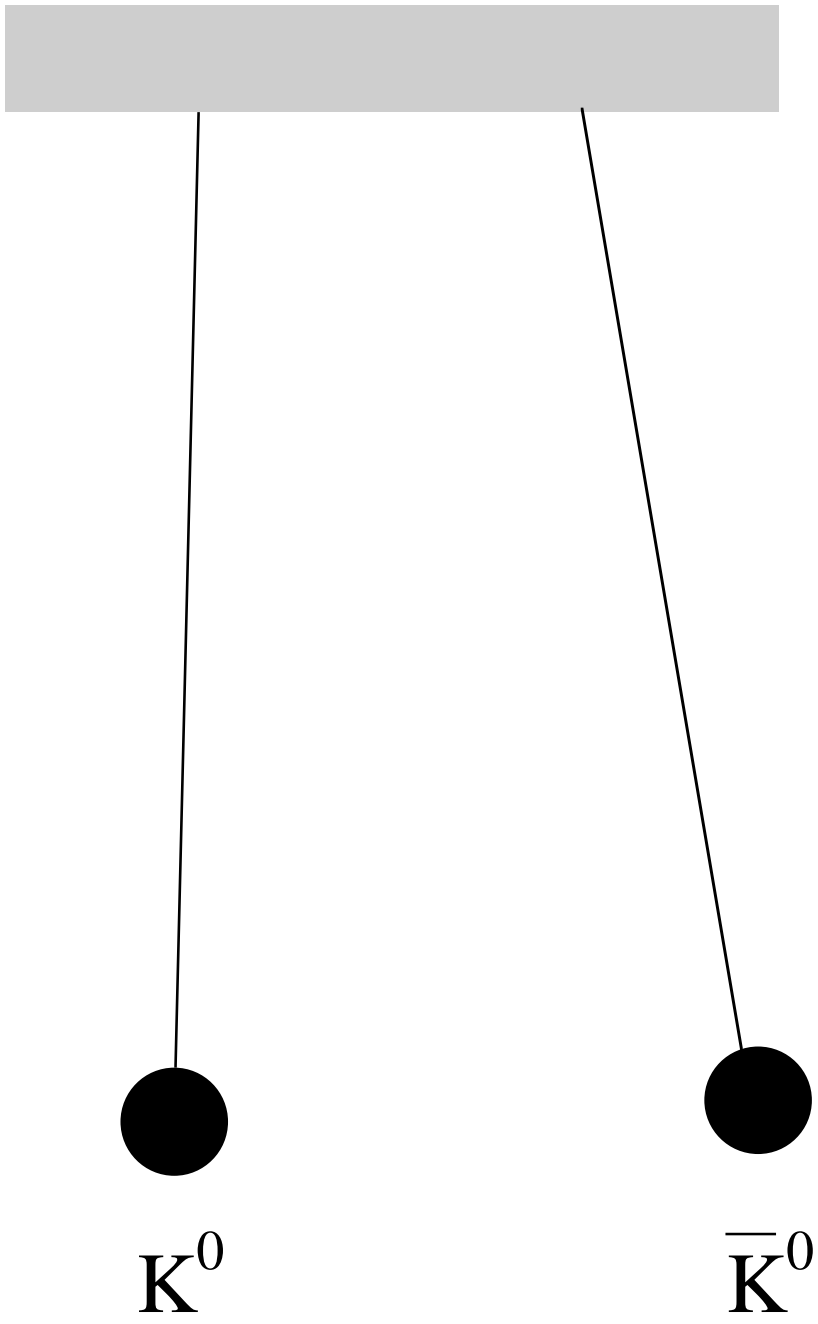
$$b(t) = \frac{1}{\Lambda_{12}}\left[i\dot{a}(t) - \Lambda_0 a(t)\right]$$

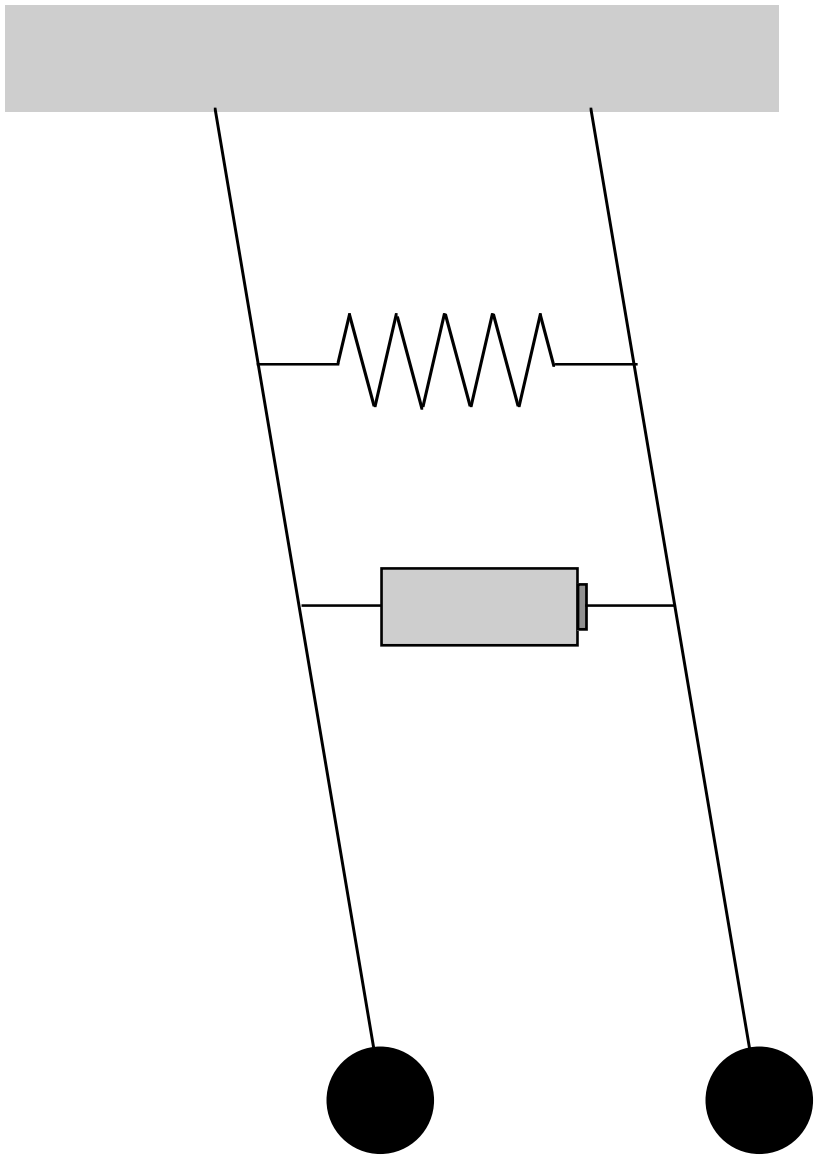
$$\Downarrow$$

$$a(t) = C_+ e^{-i\lambda_+ t} + C_- e^{-i\lambda_- t}$$

$$b(t) = \alpha\left(C_+ e^{-i\lambda_+ t} - C_- e^{-i\lambda_- t}\right), \qquad \alpha = \sqrt{\frac{\Lambda_{21}}{\Lambda_{12}}}$$

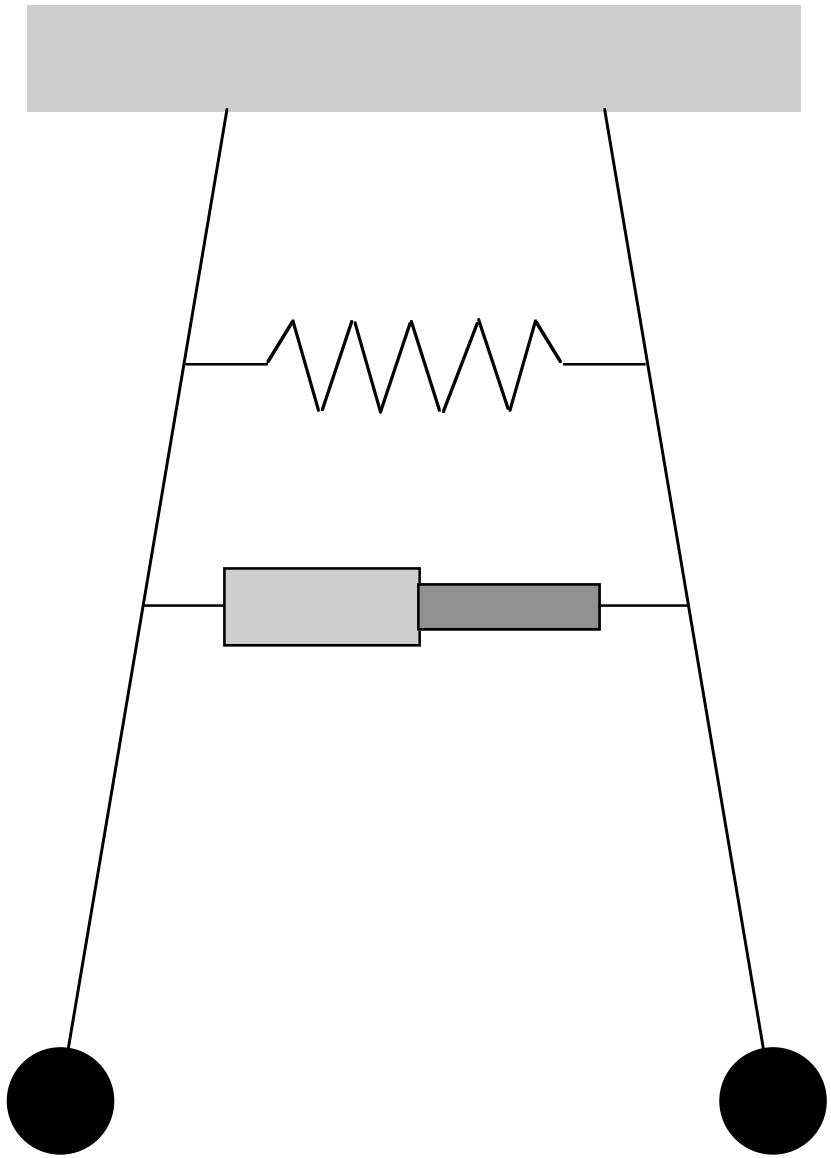
$$\lambda_\pm = \Lambda_{12} \pm \sqrt{\Lambda_{21}\Lambda_{12}} \equiv m_\pm - \frac{i}{2}\Gamma_\pm,\ m_\pm = \mathrm{Re}\,\lambda_\pm,\ \Gamma_\pm = 2\,\mathrm{Im}\,\lambda_\pm$$


K^0
$\overline{K}^0$
$\omega = m_0$



K_+

ω_+



K_-

ω_-

Two typical initial conditions:

Pure K^0 at $t = 0$

$$\left|K^0(t)\right\rangle = f_+(t)\left|K^0\right\rangle + \alpha f_-(t)\left|\overline{K}^0\right\rangle$$

$$= \frac{\sqrt{1+|\alpha|^2}}{2}\left(e^{-i\lambda_+ t}\left|K_+\right\rangle + e^{-i\lambda_- t}\left|K_-\right\rangle\right)$$

$$\left|K_\pm\right\rangle = \frac{1}{\sqrt{1+|\alpha|^2}}\left(\left|K^0\right\rangle \pm \alpha\left|\overline{K}^0\right\rangle\right)$$

$$f_\pm(t) = \frac{1}{2}\left(e^{-i\lambda_+ t} \pm e^{-i\lambda_- t}\right)$$

$K_\pm$ are **mass eigenstates**,
i.e. have definite masses and decay widths
λ_+ for K_+ and λ_- for K_-

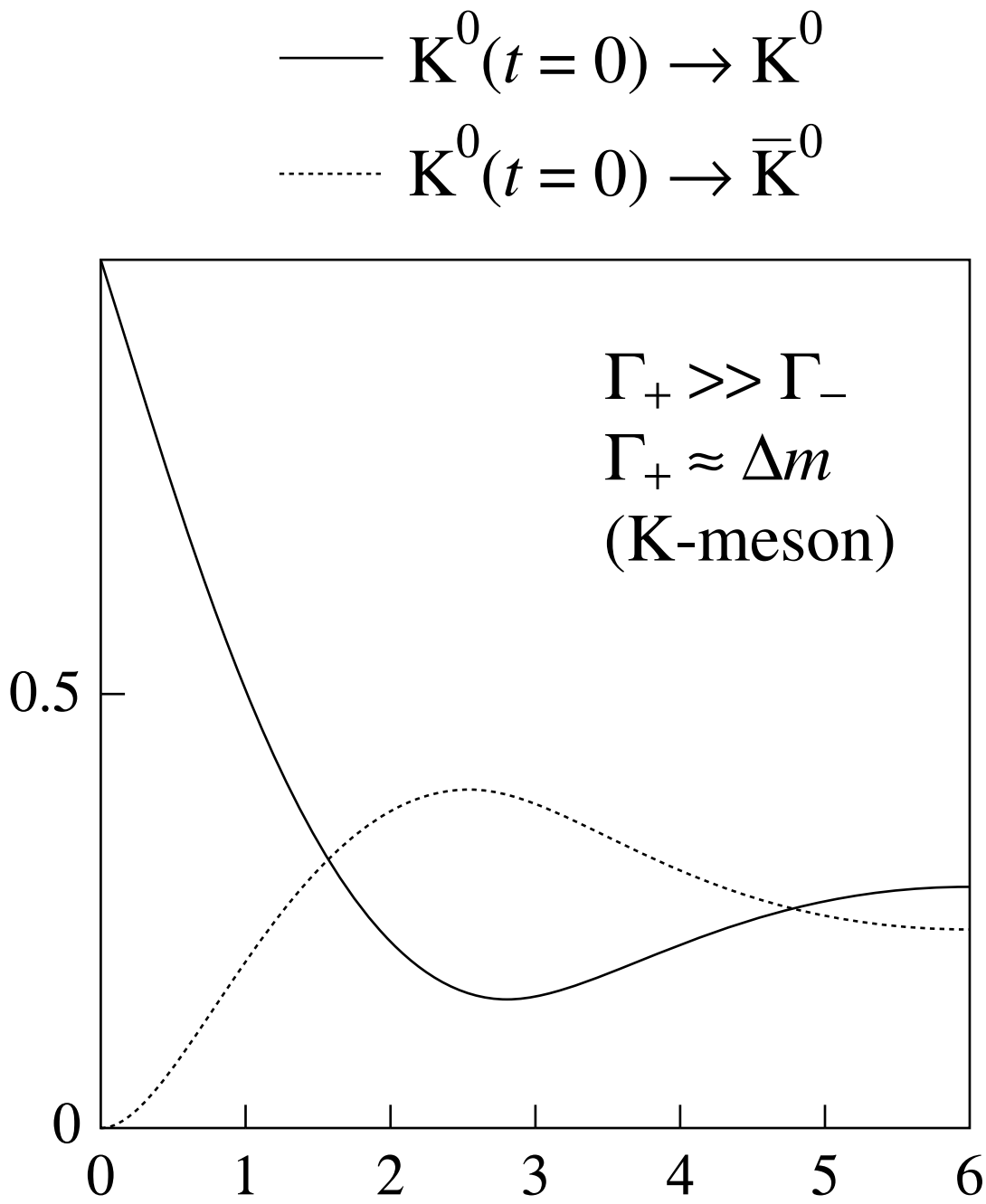
K^0 and $\overline{K}^0$ have no definite masses and decay widths:
i.e. they oscillate to each other

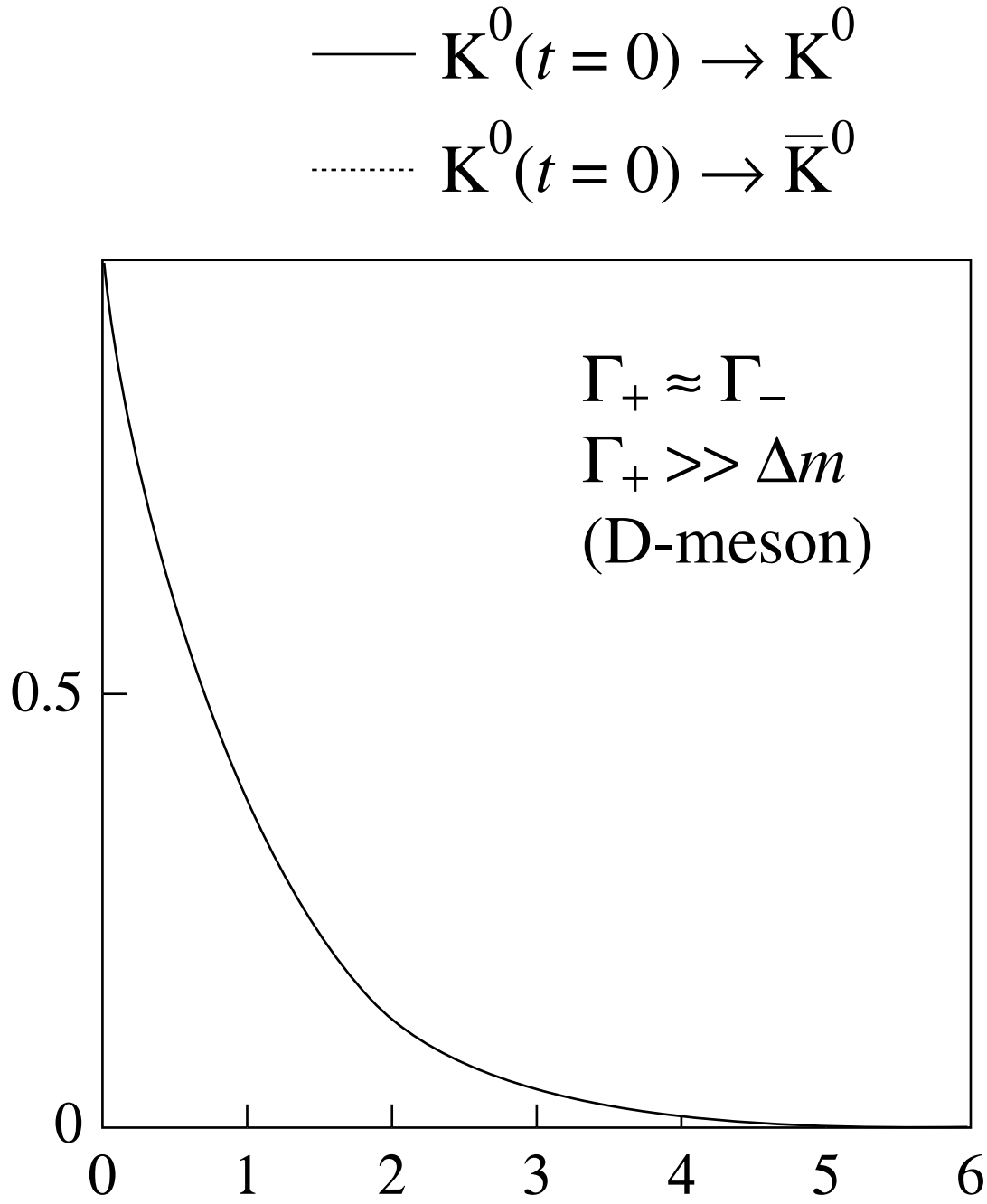
$$\text{Probability}\left[K^0(t=0) \rightarrow K^0(t)\right]$$

$$= \frac{1}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} + 2e^{-(\Gamma_+ + \Gamma_-)t/2}\cos\Delta m\, t\right]$$

$$\text{Probability}\left[K^0(t=0) \rightarrow \overline{K}^0(t)\right]$$

$$= \frac{|\alpha|^2}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-(\Gamma_+ + \Gamma_-)t/2}\cos\Delta m\, t\right]$$


$K^0(t=0) \to K^0$
$K^0(t=0) \to \bar{K}^0$
$\Gamma_+ >> \Gamma_-$
$\Gamma_+ \approx \Delta m$
(K-meson)
0.5
0
0
1
2
3
4
5
6


$K^0(t=0) \to K^0$
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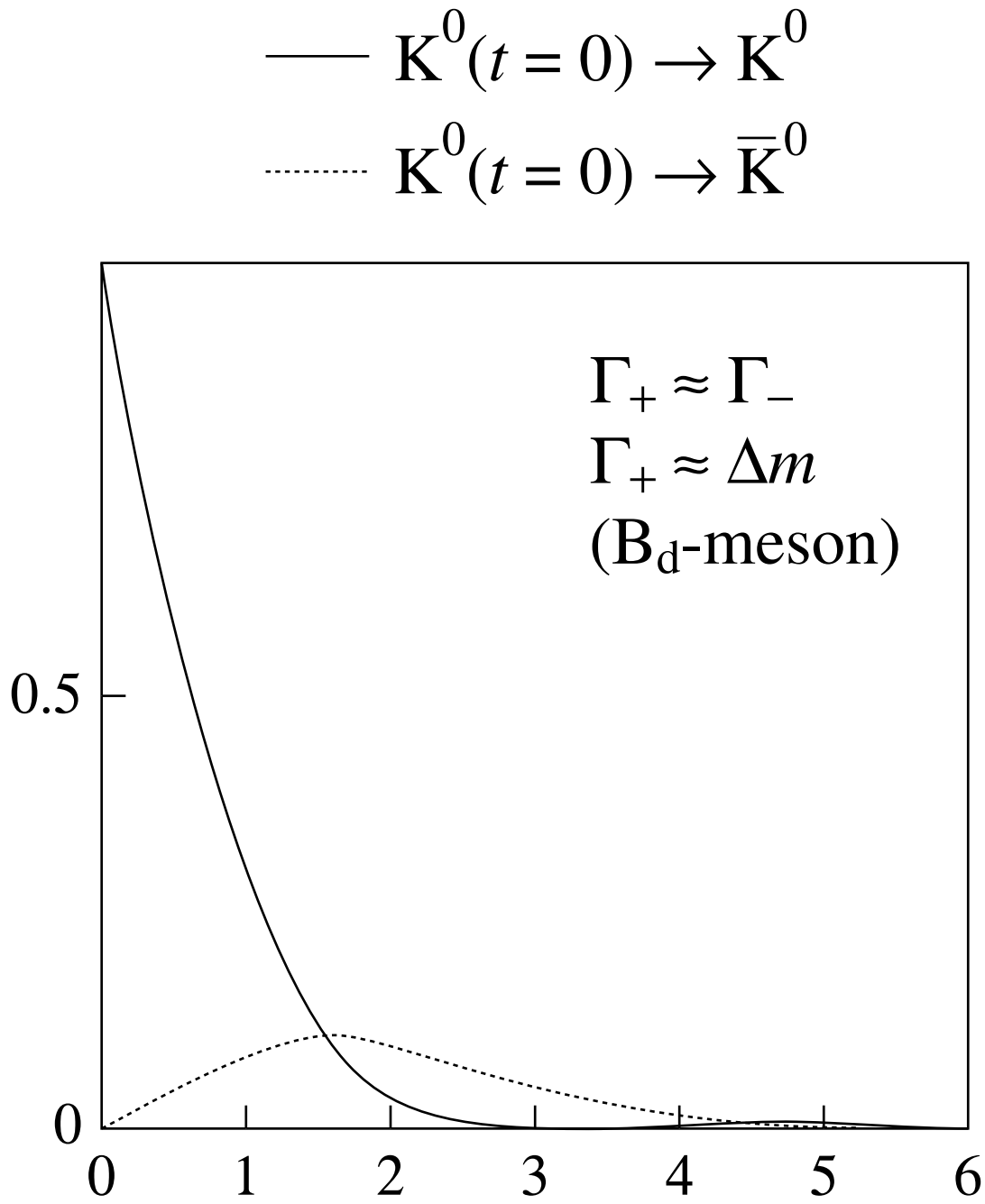
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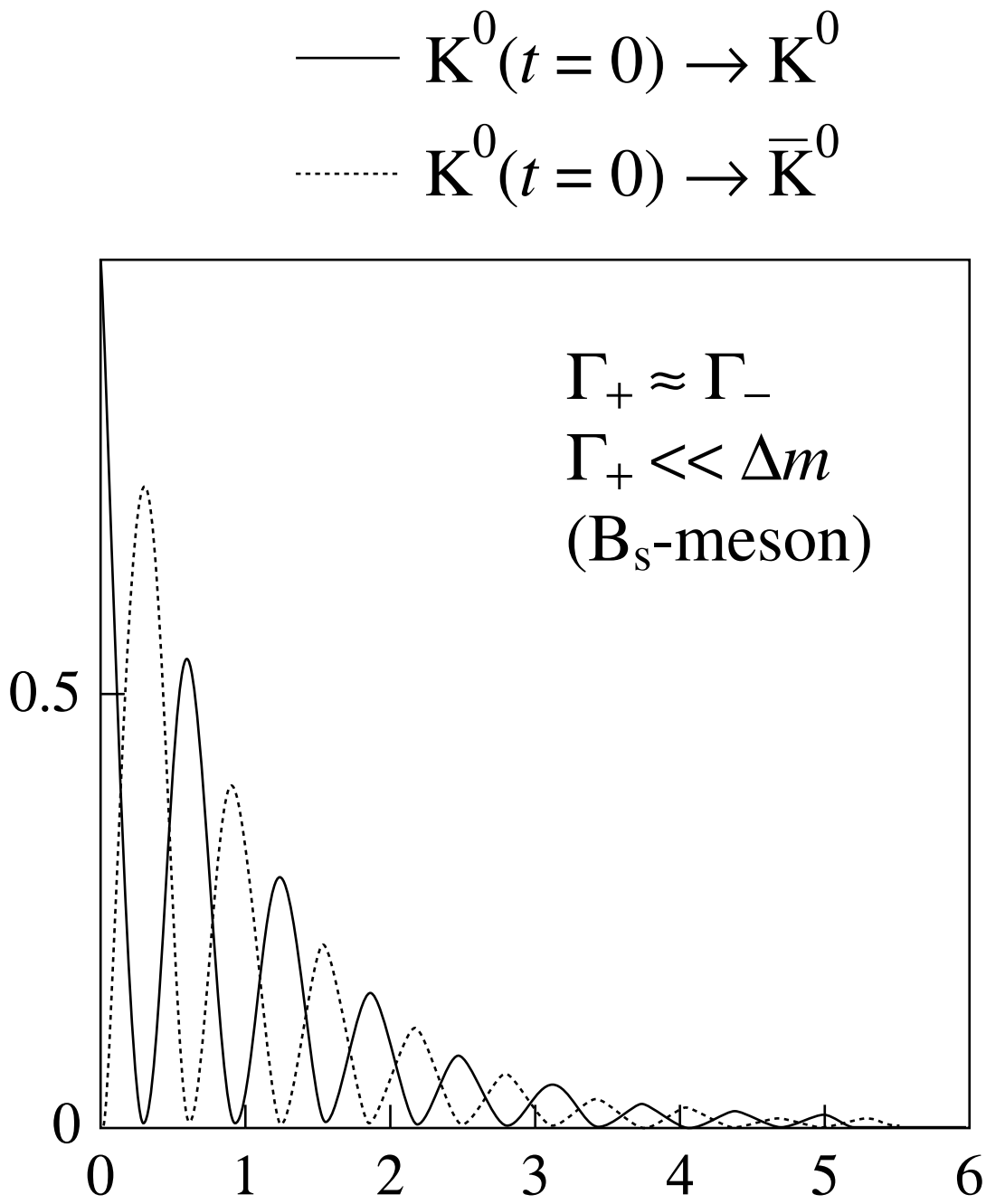

$K^0(t=0) \to K^0$
$K^0(t=0) \to \bar{K}^0$
$\Gamma_+ \approx \Gamma_-$
$\Gamma_+ << \Delta m$
(B_s-meson)
0.5
0
0
1
2
3
4
5
6

Pure $\overline{\mathrm{K}^0}$ at $t = 0$

$$\left|\overline{\mathrm{K}}^0(t)\right\rangle = \frac{1}{\alpha} f_-(t)\left|\mathrm{K}^0\right\rangle + f_+(t)\left|\overline{\mathrm{K}}^0\right\rangle$$

$$= \frac{\sqrt{1+|\alpha|^2}}{2\alpha}\left(e^{-i\lambda_+ t}\left|\mathrm{K}_+\right\rangle - e^{-i\lambda_- t}\left|\mathrm{K}_-\right\rangle\right)$$

The phases between K_+ and K_- in $|\mathrm{K}^0(t)\rangle$ and in $|\overline{\mathrm{K}}^0(t)\rangle$ are **different by π**.

$$\text{Probability}\left[\overline{\mathrm{K}}^0(\mathrm{t}=0) \rightarrow \overline{\mathrm{K}}^0(\mathrm{t})\right]$$

$$= \frac{1}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} + 2e^{-(\Gamma_+ + \Gamma_-)t/2}\cos\Delta m\, t\right]$$

$$\text{Probability}\left[\overline{\mathrm{K}}^0(\mathrm{t}=0) \rightarrow \mathrm{K}^0(\mathrm{t})\right]$$

$$= \frac{1}{4|\alpha|^2}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-(\Gamma_+ + \Gamma_-)t/2}\cos\Delta m\, t\right]$$

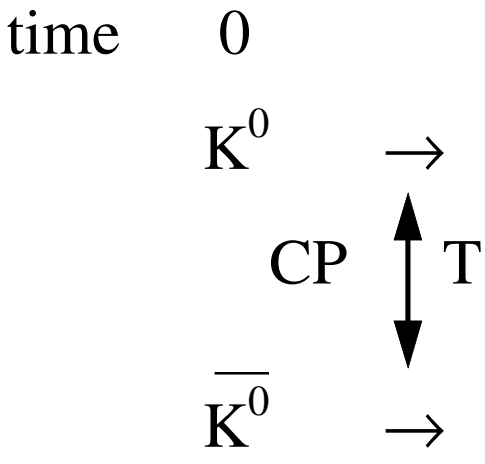
Recall, if T (and CP) is conserved

$$|\alpha| = \left|\sqrt{\frac{\Lambda_{21}}{\Lambda_{12}}}\right| = \left|\sqrt{e^{2i\varphi}}\right| = 1$$

$$\Downarrow$$

$$\text{Probability}\left[\mathrm{K}^0(\mathrm{t}=0) \to \overline{\mathrm{K}}^0(\mathrm{t})\right]$$

$$= \text{Probability}\left[\overline{\mathrm{K}}^0(\mathrm{t}=0) \to \mathrm{K}^0(\mathrm{t})\right]$$



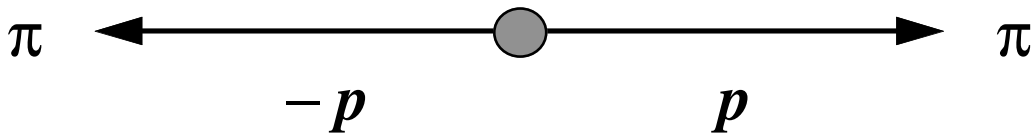
$$|\alpha| \neq 1$$

CP and T are **not conserved**

7) Assuming CP Conservation

a) Decay amplitudes

at kaon rest frame



out-going 2π states: $|2\pi>_{\text{out}}$ $\quad L=0$ $\quad CP\,|2\pi>_{\text{out}} = |2\pi>_{\text{out}}$

$$A_{2\pi} = {}_{\text{out}}\langle 2\pi | H_{\text{W}} | \text{K}^0 \rangle$$

$$= {}_{\text{out}}\langle 2\pi | (CP)^{-1}(CP) H_{\text{W}} (CP)^{-1}(CP) | \text{K}^0 \rangle$$

$$= {}_{\text{out}}\langle 2\pi | H_{\text{W}} | \overline{\text{K}}^0 \rangle e^{i\phi} \left(= \overline{A}_{2\pi} e^{i\phi} \right)$$

$$= {}_{\text{out}}\langle 2\pi | (CP)^{-1}(CP) H_{\text{W}} (CP)^{-1}(CP) | \overline{\text{K}}^0 \rangle e^{i\phi}$$

$$= {}_{\text{out}}\langle 2\pi | H_{\text{W}} | \text{K}^0 \rangle e^{i(\phi+\overline{\phi})} \left(= A_{2\pi} e^{i(\phi+\overline{\phi})} \right)$$

$$\boxed{\phi = -\overline{\phi} = \varphi}$$

b) Mass and Decay matrices

CP conservation $\quad M_{12} = M_{21} e^{-2i\varphi},\ \Gamma_{12} = \Gamma_{21} e^{-2i\varphi}$

Hermiticity $\quad M_{21} = M_{12}^{*},\ \Gamma_{21} = \Gamma_{12}^{*}$

$$\arg M_{12} = -\varphi + n\pi,\ \ \arg \Gamma_{12} = -\varphi + n'\pi$$

$$\Lambda_{ij} = M_{ij} - \frac{i}{2}\Gamma_{ij} \ \Rightarrow\ \Lambda_{12} = \Lambda_{21} e^{-2i\varphi}$$

$$\alpha = \sqrt{\frac{\Lambda_{21}}{\Lambda_{12}}} = e^{i(\varphi + n''\pi)}$$

$n,\ n',\ n''$ integer numbers

$$|\mathrm{K}_{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{K}^0\rangle \pm e^{i(\varphi+n''\pi)}|\overline{\mathrm{K}}^0\rangle\right)$$

$$CP|\mathrm{K}_{\pm}\rangle = \pm(-1)^{n''}|\mathrm{K}_{\pm}\rangle$$

$$m_{\pm} = m_0 \pm (-1)^{n+n''}|M_{12}|$$

$$\Gamma_{\pm} = \Gamma_0 \pm (-1)^{n'+n''}|\Gamma_{12}|$$

Let us take n'' to be even:

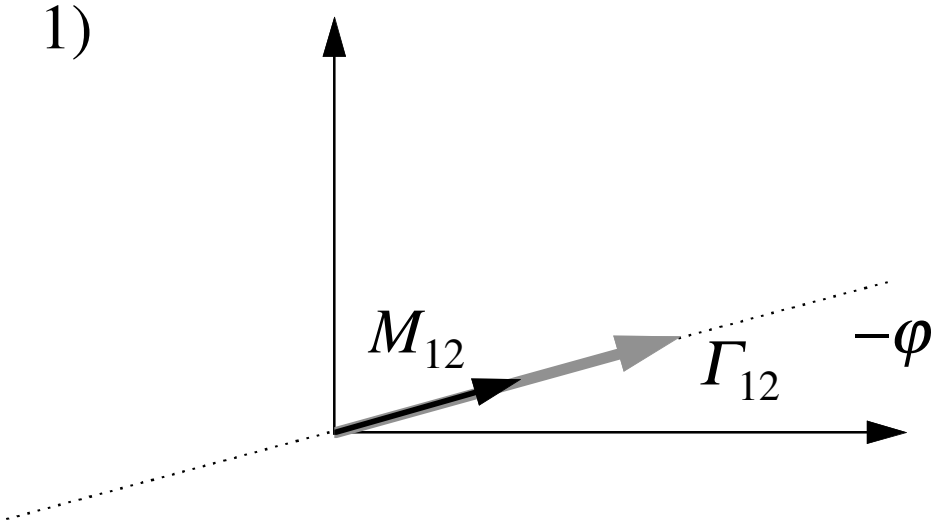
Case-1) n = even, n' = even $\quad m_{\pm} = m_0 \pm |M_{12}|, \Gamma_{\pm} = \Gamma_0 \pm |\Gamma_{12}|$
***CP* = +1 state is heavier and decay faster**

Case-2) n = even, n' = odd $\quad m_{\pm} = m_0 \pm |M_{12}|, \Gamma_{\pm} = \Gamma_0 \mp |\Gamma_{12}|$
***CP* = +1 state is heavier and decay slower**

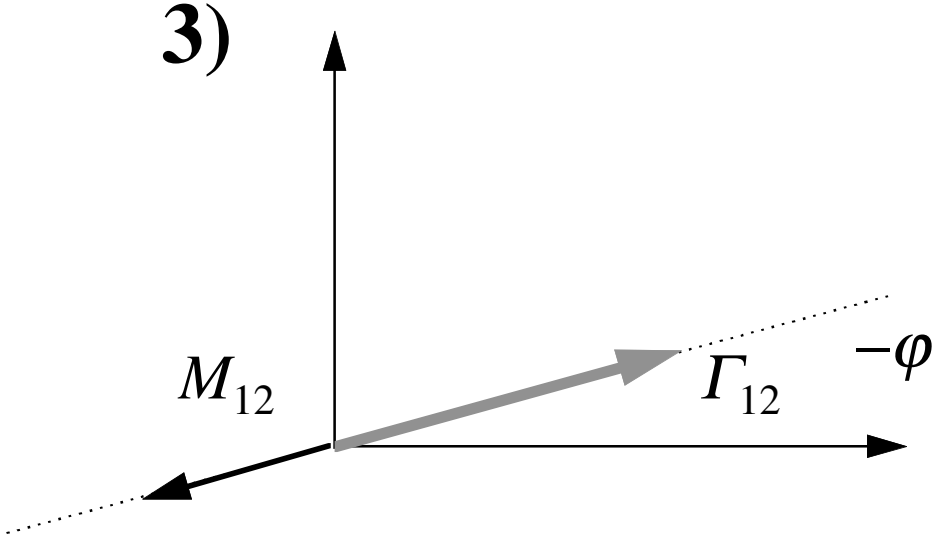
Case-3) n = odd, n' = even $\quad m_{\pm} = m_0 \mp |M_{12}|, \Gamma_{\pm} = \Gamma_0 \pm |\Gamma_{12}|$
***CP* = +1 state is lighter and decay faster**

Case-4) n = odd, n' = odd $\quad m_{\pm} = m_0 \mp |M_{12}|, \Gamma_{\pm} = \Gamma_0 \mp |\Gamma_{12}|$
***CP* = +1 state is lighter and decay slower**

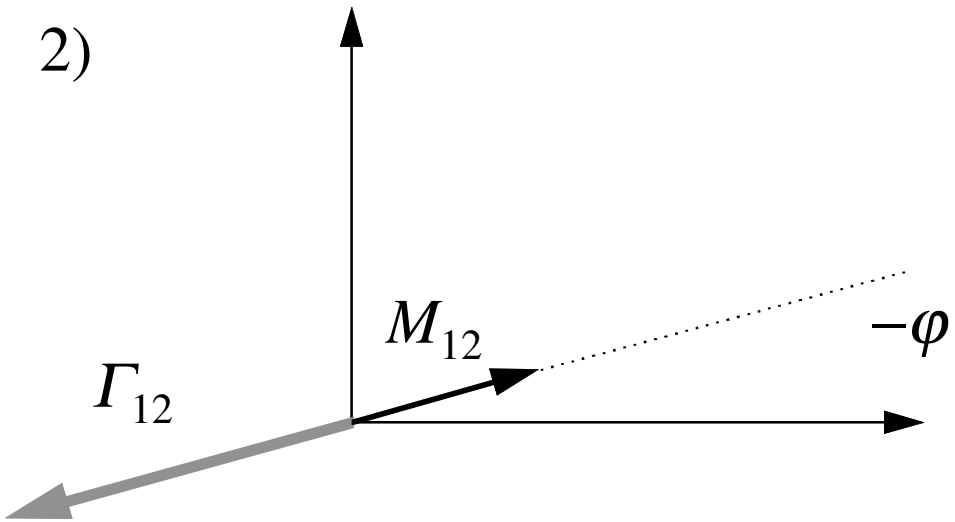
1)


M_{12}
Γ_{12}
$-\varphi$

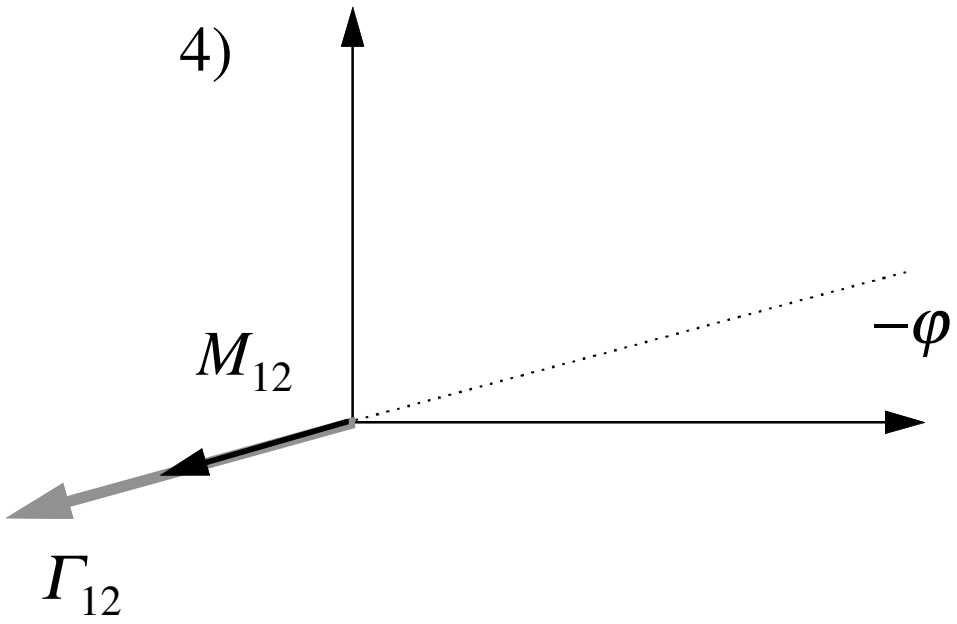
3)


M_{12}
Γ_{12}
$-\varphi$

2)


M_{12}
Γ_{12}
$-\varphi$

4)


M_{12}
Γ_{12}
$-\varphi$

If we take n'' to be odd:

Case-1) n = even, n' = even $\quad m_\pm = m_0 \mp |M_{12}|, \Gamma_\pm = \Gamma_0 \mp |\Gamma_{12}|$

CP = +1 state is heavier and decay faster

Case-2) n = even, n' = odd $\quad m_\pm = m_0 \mp |M_{12}|, \Gamma_\pm = \Gamma_0 \pm |\Gamma_{12}|$

CP = +1 state is heavier and decay slower

Case-3) n = odd, n' = even $\quad m_\pm = m_0 \pm |M_{12}|, \Gamma_\pm = \Gamma_0 \mp |\Gamma_{12}|$

CP = +1 state is lighter and decay faster

Case-4) n = odd, n' = odd $\quad m_\pm = m_0 \pm |M_{12}|, \Gamma_\pm = \Gamma_0 \pm |\Gamma_{12}|$

CP = +1 state is lighter and decay slower

identical to n'' = even !!!

The choice of n'' is **irrelevant**: ***let us take to be even***

What do we know about K_S and K_L? (PDG 1996)

$$\tau_S = \frac{1}{\Gamma_S} = (0.8927 \pm 0.0009) \times 10^{-10}\,\mathrm{s}$$

$$\tau_L = \frac{1}{\Gamma_L} = (5.17 \pm 0.04) \times 10^{-8}\,\mathrm{s}$$

$$\Delta m = m_L - m_S = (0.5304 \pm 0.0014) \times 10^{10}\,\hbar\mathrm{s}^{-1}$$

$K_S \to 2\pi$ and almost no $K_L \to 2\pi$

Case-3 should be applied to the neutral kaon system !!

$$\frac{\Delta m}{\Delta\Gamma} = \frac{|M_{12}|}{|\Gamma_{12}|} = 0.4743 \pm 0.0012 \quad \left(\Delta\Gamma = \Gamma_S - \Gamma_L\right)$$

8) Including Small CP Violation

$$\arg M_{12} = -\varphi + \pi + \xi, \ \arg \Gamma_{12} = -\varphi + \xi'$$

small CP violation:

$$|\xi| << 1 \text{ and } |\xi'| << 1, \text{ thus } |\Delta_{\text{M-}\Gamma}| \equiv |\xi - \xi'| << 1$$

$$\alpha = (1 - 2\varepsilon_{\text{T}})e^{-i\varphi_\Gamma}$$

$$\varepsilon_{\text{T}} = \frac{r\Delta_{\text{M-}\Gamma}}{4r^2 + 1}(1 + i2r), \ r = \frac{|M_{12}|}{|\Gamma_{12}|} = \frac{\Delta m}{\Delta\Gamma} = 0.4743 \pm 0.0012$$

r is not related to *CP* violation, i.e. phase of ε_{T} is defined.
$\Delta_{\text{M-}\Gamma}$ is only the unknown parameter for *CP* violation

$$\varphi_\Gamma = \arg \Gamma_{12} : \text{ usually fixed by a phase convention}$$

$$\arg \varepsilon_{\text{T}} = \tan^{-1} \frac{2\Delta m}{\Delta\Gamma} = 43.49^\circ \pm 0.07^\circ : \text{ superweak phase}$$

Mass eigenstates are given by:

$$|\mathrm{K_S}\rangle = \frac{1}{\sqrt{2}}\left[|\mathrm{K}^0\rangle + (1-2\varepsilon_\mathrm{T})e^{-i\varphi_\Gamma}|\overline{\mathrm{K}}^0\rangle\right]$$

$$|\mathrm{K_L}\rangle = \frac{1}{\sqrt{2}}\left[|\mathrm{K}^0\rangle - (1-2\varepsilon_\mathrm{T})e^{-i\varphi_\Gamma}|\overline{\mathrm{K}}^0\rangle\right]$$

$$\lambda_\mathrm{S} = (m_0 - |M_{12}|) - \frac{i}{2}(\Gamma_0 + |\Gamma_{12}|)$$

$$\lambda_\mathrm{L} = (m_0 + |M_{12}|) - \frac{i}{2}(\Gamma_0 - |\Gamma_{12}|)$$

Observed CP violation in the K_L semileptonic decays

$$K_L \to \ell^+ \pi^- \nu \xleftrightarrow{CP} K_L \to \ell^- \pi^+ \bar{\nu} \ :$$

if CP is conserved

Charge asymmetry in semileptonic K_L decays;

$$\delta = \frac{\Gamma\left(K_L \to \ell^+ \pi^- \nu\right) - \Gamma\left(K_L \to \ell^- \pi^+ \nu\right)}{\Gamma\left(K_L \to \ell^+ \pi^- \nu\right) + \Gamma\left(K_L \to \ell^- \pi^+ \nu\right)} \qquad \text{(PDG, 1996)}$$

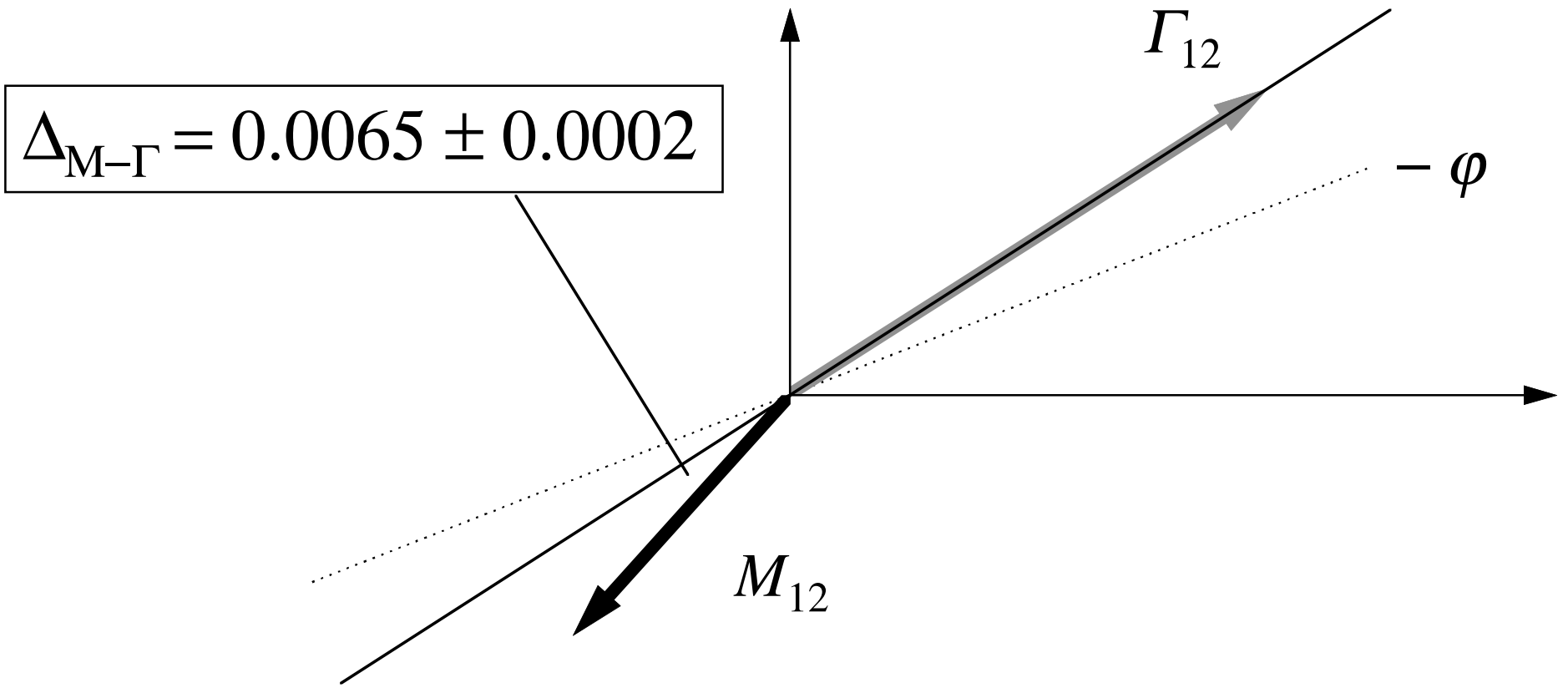
$$= 2\,\mathrm{Re}\,\varepsilon_T = \frac{2r}{4r^2+1}\Delta_{M\text{-}\Gamma} = (3.27 \pm 0.12) \times 10^{-3}$$

assuming

- $\Delta Q = \Delta S$
- no CP violation in the semileptonic decay amplitudes

$$\left|\left\langle \ell^+ \pi^- \nu \middle| H_W \middle| K^0 \right\rangle\right| = \left|\left\langle \ell^- \pi^+ \bar{\nu} \middle| H_W \middle| \bar{K}^0 \right\rangle\right|$$

M_{12} and Γ_{12} are **not** perfectly **antiparallel**
$\rightarrow$ generating **CP** (and T) **violation**



Another CP violation in the semileptonic decays:

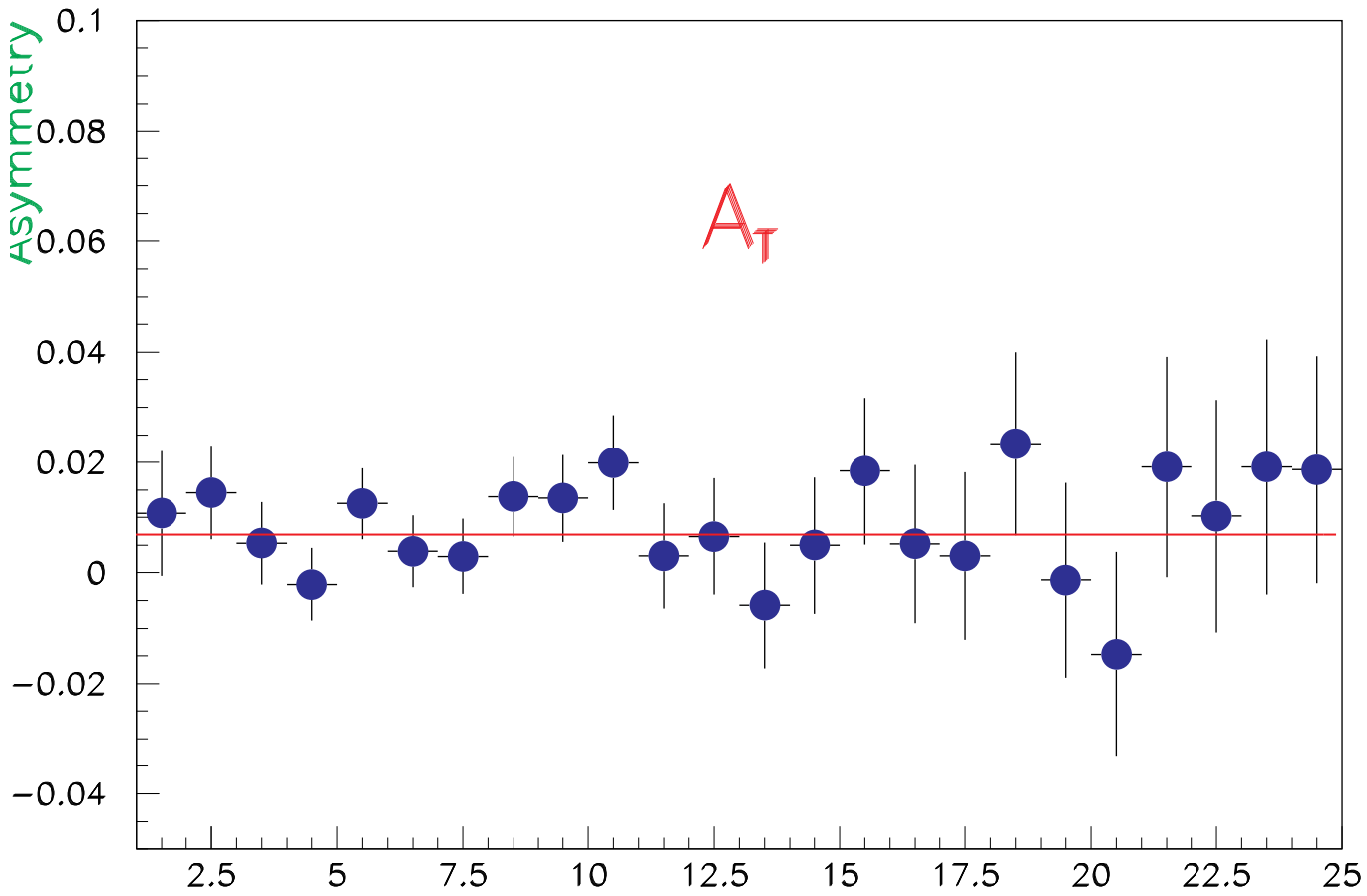
$$A_T(t) = \frac{\Gamma\left[\mathrm{K}^0_{t=0} \to \overline{\mathrm{K}}^0(\mathrm{t})\right] - \Gamma\left[\overline{\mathrm{K}}^0_{t=0} \to \mathrm{K}^0(\mathrm{t})\right]}{\Gamma\left[\mathrm{K}^0_{t=0} \to \overline{\mathrm{K}}^0(\mathrm{t})\right] + \Gamma\left[\overline{\mathrm{K}}^0_{t=0} \to \mathrm{K}^0(\mathrm{t})\right]}$$

$$= \frac{\Gamma\left[\mathrm{K}^0_{t=0} \to \ell^- \pi^+ \overline{\nu}(\mathrm{t})\right] - \Gamma\left[\overline{\mathrm{K}}^0_{t=0} \to \ell^+ \pi^- \nu(\mathrm{t})\right]}{\Gamma\left[\mathrm{K}^0_{t=0} \to \ell^- \pi^+ \overline{\nu}(\mathrm{t})\right] + \Gamma\left[\overline{\mathrm{K}}^0_{t=0} \to \ell^+ \pi^- \nu(\mathrm{t})\right]}$$

$$= 4\,\mathrm{Re}\,\varepsilon_{\mathrm{T}}$$

(discussed previously)

CPLEAR


Asymmetry
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.04
A_T
2.5
5
7.5
10
12.5
15
17.5
20
22.5
25