9) CP Violation in the decays into two pions

a) Two pion final states

$|\pi\pi>_{\text{out}} \rightarrow$ eigenstate of strong interactions

can be expanded with isospin eigenstates

$$\begin{array}{ccc} \text{K} & \rightarrow & \pi\pi \\ s = 0 & & \ell = 0 \\ & & I = 0,\, 2 \end{array} \qquad \text{(Bose-Einstein statistics)}$$

$$\left|\pi^+\pi^-\right\rangle_{\text{out}} = \sqrt{\frac{2}{3}}|\, I = 0\rangle_{\text{out}} + \sqrt{\frac{1}{3}}|\, I = 2\rangle_{\text{out}}$$

$$\left|\pi^0\pi^0\right\rangle_{\text{out}} = -\sqrt{\frac{1}{3}}|\, I = 0\rangle_{\text{out}} + \sqrt{\frac{2}{3}}|\, I = 2\rangle_{\text{out}}$$

b) Relation between K and $\overline{\mathrm{K}}$ decay amplitudes

$$\overline{A}_0 = {}_{\mathrm{out}}\langle 2\pi(I=0)|H_W\left|\overline{\mathrm{K}}^0\right\rangle$$

$$= {}_{\mathrm{out}}\langle 2\pi(I=0)|(CPT)^{-1}(CPT)H_W(CPT)^{-1}(CPT)\left|\overline{\mathrm{K}}^0\right\rangle$$

$$= \left\{{}_{\mathrm{in}}\langle 2\pi(I=0)|H_W\left|\mathrm{K}^0\right\rangle\right\}^* e^{-i(\overline{\theta}-\varphi)}$$

S-matrix is unitary
and
diagonal at the kaon energy

$2\pi(I=0)$	$\rightarrow$	$2\pi(I=0)$
	$\not\rightarrow$	$n\pi(I=0)$

$$\left|2\pi(I=0)\right\rangle_{\mathrm{in}} = S^{-1}\left|2\pi(I=0)\right\rangle_{\mathrm{out}}$$

$$= e^{2i\delta_0}\left|2\pi(I=0)\right\rangle_{\mathrm{out}}$$

δ_0: π-π (I= 0) scattering phase shift

$$\begin{aligned}
\overline{A}_0 &= {}_{\text{out}}\langle 2\pi(I=0)|H_W\left|\overline{\mathrm{K}}^0\right\rangle \\
&= \left\{{}_{\text{in}}\langle 2\pi(I=0)|H_W\left|\mathrm{K}^0\right\rangle\right\}^* e^{-i(\overline{\theta}-\varphi)} \\
&= \left\{{}_{\text{out}}\langle 2\pi(I=0)|H_W\left|\mathrm{K}^0\right\rangle\right\}^* e^{2i\delta_0} e^{-i(\overline{\theta}-\varphi)} \\
&= A_0^* e^{2i\delta_0} e^{-i(\overline{\theta}-\varphi)}
\end{aligned}$$

$A_0 \to a_0 e^{i\delta_0}$	$A_2 \to a_2 e^{i\delta_2}$
$\overline{A}_0 \to a_0^* e^{i\delta_0} e^{i(\varphi-\overline{\theta})}$	$\overline{A}_2 \to a_2^* e^{i\delta_2} e^{i(\varphi-\overline{\theta})}$

c) $\Delta I = 1/2$ rule

$$A_{+-} = \sqrt{\frac{2}{3}}A_0 + \sqrt{\frac{1}{3}}A_2$$

$$A_{00} = -\sqrt{\frac{1}{3}}A_0 + \sqrt{\frac{2}{3}}A_2$$

if A_2=0

$$\frac{\Gamma\left(\mathrm{K_S} \to \pi^0\pi^0\right)}{\Gamma\left(\mathrm{K_S} \to \pi^+\pi^-\right)} = \frac{|A_{00}|^2}{|A_{+-}|^2} = \frac{1}{2}$$

Experiment ~0.46

In reality $\dfrac{|A_2|}{|A_0|} \approx 0.045$

K	2π	
$I = 1/2$	$I = 0$	$\Delta I = 1/2$ ←
	$I = 2$	$\Delta I = 3/2$

d) CP violation in $K \to 2\pi$

if CP is conserved:

$$K_S\,(CP = +1) \to 2\pi\,(CP = +1)$$

$$K_L\,(CP = -1) \not\to 2\pi\,(CP = +1)$$

CP violating decay modes

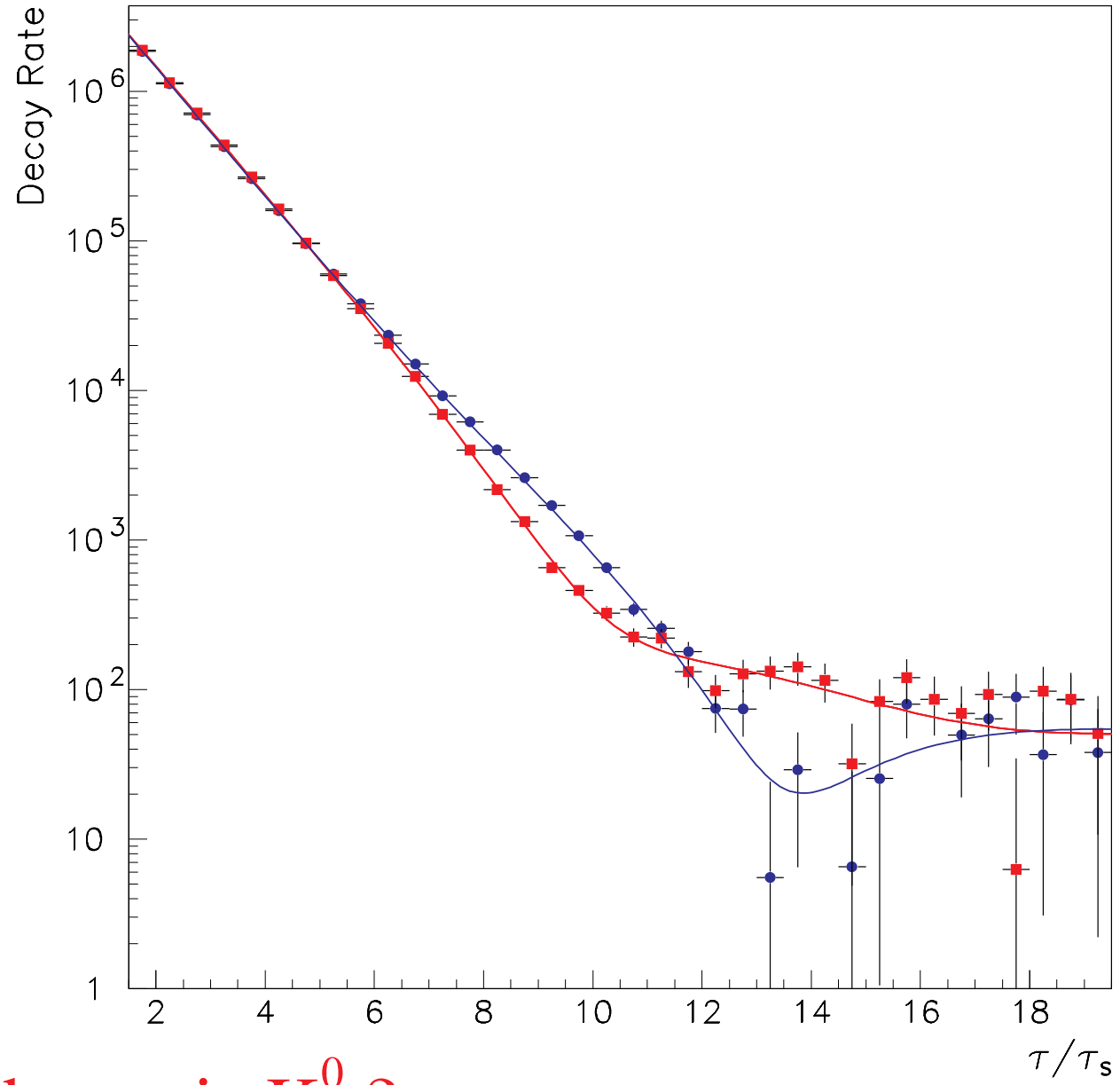
(This is how CP violation was discovered !!)

or

$$K^0_{t=0} \to 2\pi(t) = \overline{K}^0_{t=0} \to 2\pi(t)$$

CP symmetries

CPLEAR



Which one is K^0 ?

initially pure K^0

$$\left|K^0(t)\right\rangle = f_+(t)\left|K^0\right\rangle + (1-2\varepsilon_T)e^{-i\varphi_\Gamma} f_-(t)\left|\overline{K}^0\right\rangle$$

$$= \frac{1}{\sqrt{2}}\left[e^{-i\lambda_S t}\left|K_S\right\rangle + e^{-i\lambda_L t}\left|K_L\right\rangle\right]$$

time dependent decay rate

K_S and $K_L \to 2\pi$ decays can interfere

$$\Gamma_{+-}(t) = \frac{1}{2}\left|e^{-i\lambda_S t} A^S_{+-} + e^{-i\lambda_L t} A^L_{+-}\right|^2$$

$$= \frac{\left|A^S_{+-}\right|^2}{2}\left[e^{-\Gamma_S t} + \left|\frac{A^S_{+-}}{A^S_{+-}}\right|^2 e^{-\Gamma_L t} + 2\left|\frac{A^S_{+-}}{A^S_{+-}}\right| e^{-\overline{\Gamma} t}\cos(\Delta m t - \varphi_{+-})\right]$$

$$A^S_{+-} = \frac{1}{\sqrt{2}} A_{+-} + (1-2\varepsilon_T)e^{-i\varphi_\Gamma}\overline{A}_{+-}$$

$$A^L_{+-} = \frac{1}{\sqrt{2}} A_{+-} - (1-2\varepsilon_T)e^{-i\varphi_\Gamma}\overline{A}_{+-}$$

$$\varphi_{+-} = \arg\left(\frac{A^L_{+-}}{A^S_{+-}}\right)$$

initially pure $\overline{\mathrm{K}}^0$

$$\left|\overline{\mathrm{K}}^0(t)\right\rangle = (1+2\varepsilon_{\mathrm{T}})e^{i\varphi_\Gamma} f_-(t)\left|\mathrm{K}^0\right\rangle + f_+(t)\left|\overline{\mathrm{K}}^0\right\rangle$$

$$= \frac{(1+2\varepsilon_{\mathrm{T}})e^{i\varphi_\Gamma}}{\sqrt{2}}\left[e^{-i\lambda_{\mathrm{S}}t}\left|\mathrm{K_S}\right\rangle - e^{-i\lambda_{\mathrm{L}}t}\left|\mathrm{K_L}\right\rangle\right]$$

time dependent decay rate

$$\overline{\Gamma}_{+-}(t) = \frac{1+4\,\mathrm{Re}\,\varepsilon_{\mathrm{T}}}{2}\left|e^{-i\lambda_{\mathrm{S}}t}A^{\mathrm{S}}_{+-} - e^{-i\lambda_{\mathrm{L}}t}A^{\mathrm{L}}_{+-}\right|^2$$

$$\propto \left[e^{-\Gamma_{\mathrm{S}}t} + \left|\frac{A^{\mathrm{L}}_{+-}}{A^{\mathrm{S}}_{+-}}\right|^2 e^{-\Gamma_{\mathrm{L}}t} - 2\left|\frac{A^{\mathrm{L}}_{+-}}{A^{\mathrm{S}}_{+-}}\right| e^{-\overline{\Gamma}t}\cos(\Delta m t - \varphi_{+-})\right]$$

initial-K^0 and initial-$\overline{\mathrm{K}}0$

$\mathrm{K_S}$ - $\mathrm{K_L}$ interference terms have opposite sign !!

CP violation parameter

$$\eta_{+-} \equiv \frac{A^{\mathrm{L}}_{+-}}{A^{\mathrm{S}}_{+-}}$$ **= 0 if CP is conserved**

$$\varphi_{+-} \equiv \arg \eta_{+-}$$

similarly for $\mathrm{K} \to \pi^0\pi^0$ decays

$$\eta_{00} \equiv \frac{A^{\mathrm{L}}_{00}}{A^{\mathrm{S}}_{00}}$$

$$\varphi_{00} \equiv \arg \eta_{00}$$

e) CP violation in K→2π(*I*=0) and 2π(*I*=2)

let assume we can measure directly 2π(*I*=0) and 2π(*I*=2) states

2π(*I*=0) state

$$\eta_0 \equiv \frac{A_0^{\mathrm{L}}}{A_0^{\mathrm{S}}}$$

$$= \frac{a_0 - (1-2\varepsilon_{\mathrm{T}})e^{-i\varphi_\Gamma} a_0^* e^{i(\varphi-\bar{\theta})}}{a_0 + (1-2\varepsilon_{\mathrm{T}})e^{-i\varphi_\Gamma} a_0^* e^{i(\varphi-\bar{\theta})}}$$

$$= \varepsilon_{\mathrm{T}} - \frac{i}{2}(\varphi - \bar{\theta} - 2\varphi_{I=0} - \varphi_\Gamma)$$

CP and T phases

$\varphi_{I=0} = \arg a_0$

$\varphi_\Gamma = \arg \Gamma_{12}$

Phase of Γ_{12}

$$\Gamma_{ij} = 2\pi \sum_f \langle i|H_{\rm W}|f\rangle\langle f|H_{\rm W}|j\rangle \delta\left(m_0 - E_f\right)$$

$$\propto A_0^* \overline{A}_0 + A_2^* \overline{A}_2 + A_{3\pi}^* \overline{A}_{3\pi}$$

$$\approx A_0^* \overline{A}_0 \qquad \Delta I = 1/2 \text{ rule !!}$$

$$\propto e^{i\left(\varphi - \bar{\theta} - 2\varphi_{I=0}\right)} \longrightarrow \textbf{This is a very good approximation.}$$

$$\eta_0 = \varepsilon_{\rm T} - \frac{i}{2}\left(\varphi - \bar{\theta} - 2\varphi_{I=0} - \varphi_\Gamma\right)$$

$$\approx \varepsilon_{\rm T}$$

$2\pi(I=2)$ state

$$\eta_2 \equiv \frac{A_2^{\mathrm{L}}}{A_2^{\mathrm{S}}}$$

$$= \varepsilon_{\mathrm{T}} - \frac{i}{2}\left(\varphi - \bar{\theta} - 2\varphi_{I=2} - \varphi_{\Gamma}\right)$$

$$= \varepsilon_{\mathrm{T}} - i\left(\varphi_{I=0} - \varphi_{I=2}\right)$$

$\varphi_{I=0} = \arg a_0$ $\qquad$ $\varphi_{I=2} = \arg a_2$

If weak phases between $I = 0$ and $I = 2$ decay amplitudes are different

$$\eta_{\mathrm{I=0}} \neq \eta_{\mathrm{I=2}}$$

f) CP violation in $K \to \pi^+\pi^-$ and $\pi^0\pi^0$

$$\eta_{+-} \equiv \frac{A^{\mathrm{L}}_{+-}}{A^{\mathrm{S}}_{+-}}$$

$$\approx \eta_0 + \frac{\eta_2 - \eta_0}{\sqrt{2}} \frac{A^{\mathrm{S}}_2}{A^{\mathrm{S}}_0}$$

suppressed by the $\Delta I = 1/2$ rule

$$= \varepsilon_{\mathrm{T}} + \frac{\eta_2 - \eta_0}{\sqrt{2}} \frac{A^{\mathrm{S}}_2}{A^{\mathrm{S}}_0}$$

usually referred as ε'

$\varepsilon' \neq 0$ only if there exists a phase difference between a_0 and a_2

$$\varepsilon' \equiv \frac{\eta_2 - \eta_0}{\sqrt{2}} \frac{A_2^{\mathrm{S}}}{A_2^{\mathrm{L}}}$$

$$= \frac{1}{\sqrt{2}} \mathrm{Im} \frac{a_2}{a_0} e^{i(\pi/2 + \delta_2 - \delta_0)}$$

$$\boxed{\eta_{+-} \approx \varepsilon_T + \varepsilon', \quad \eta_{00} \approx \varepsilon_T - 2\varepsilon'}$$

10) Standard model Description of CP Violation

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix}$$

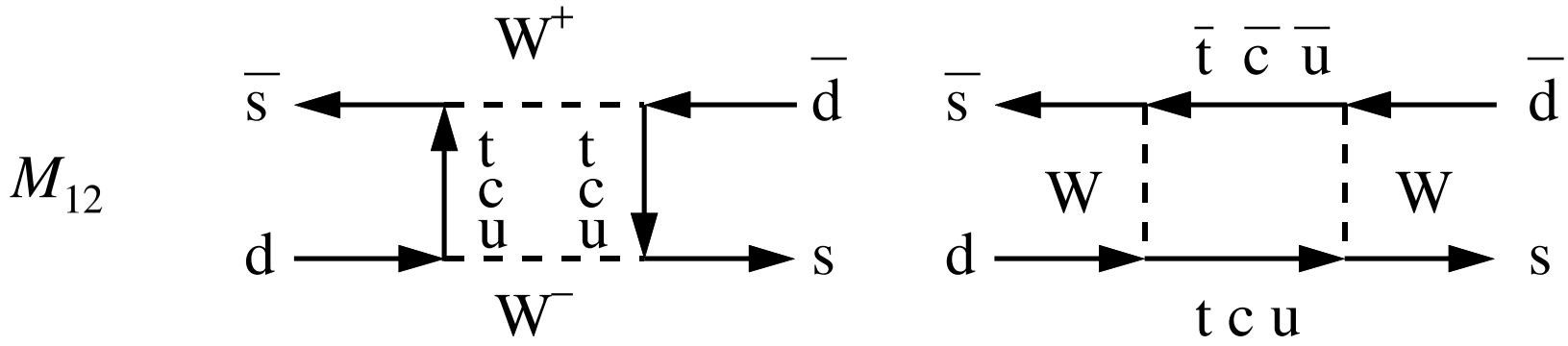
V_{ud}, V_{us}, V_{cs}, V_{cb}, V_{tb} are (practically) real

V_{cd}, V_{ts} could be slightly complex

V_{td}, V_{ub} could be complex

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda\left[1 + A^2\lambda^4(\rho + i\eta)\right] & \left(1-\frac{\lambda^2}{2}\right) & A\lambda^2 \\ A\lambda^3\left[1-(\rho+i\eta)\left(1-\frac{\lambda^2}{2}\right)\right] & -A\lambda^2\left[\left(1-\frac{\lambda^2}{2}\right)+\lambda^2(\rho+i\eta)\right] & 1 \end{pmatrix}$$

$A \sim 1, \lambda \sim 0.22, \rho \neq 0$ but $\eta \neq 0$???



very complex: t-$\bar{t}$, $(V_{ts}\,V_{td}^{*})^2 \sim \lambda^{10} \times m_t^2$

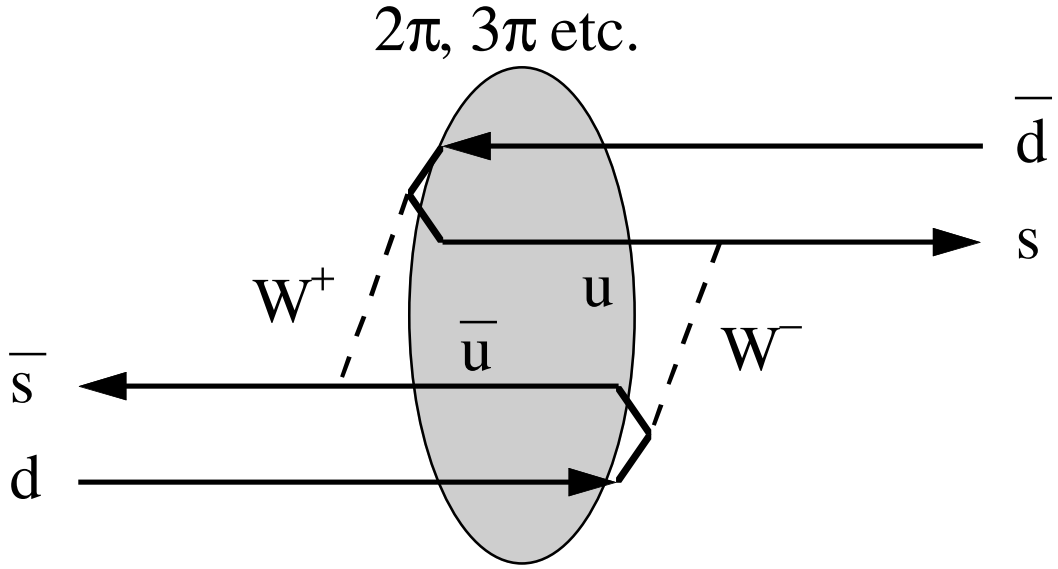
complex: t-$\bar{c}$+c-$\bar{t}$ $2V_{ts}\,V_{td}^{*}\,V_{cs}\,V_{cd}^{*} \sim 2 \times \lambda^{6} \times m_t\, m_c$

only slightly complex:c-$\bar{c}$, $(V_{cs}\,V_{cd}^{*})^2 \sim \lambda^{2} \times m_c^2$

dominating term

M_{12} is **only slightly** complex even $\eta \sim 1$

Γ_{12}

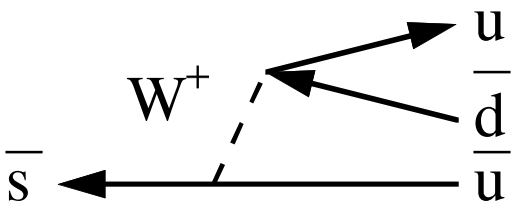


real

Then M_{12} only slightly complex and Γ_{12} real $\rightarrow \varepsilon_T \neq 0$

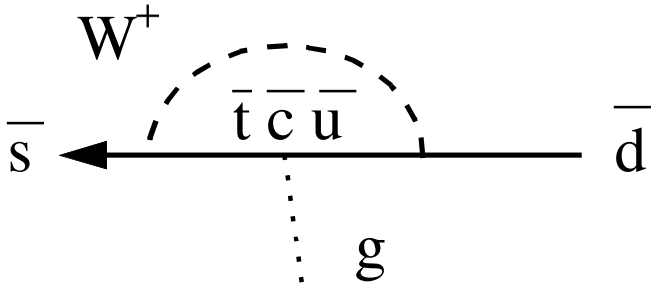
but $\Delta_{M\text{-}\Gamma}$ is very small

$I = 0$ and $I = 2$



real

$I = 0$

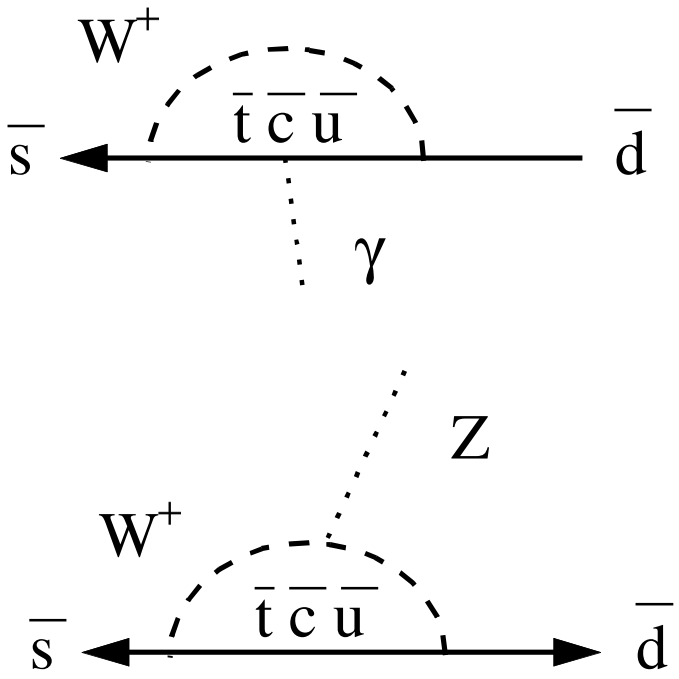


complex

A_0 is complex and A_2 is real $\rightarrow \varepsilon' \neq 0$

In reality, theoretical calculation of ε' is extremely difficult, i.e. large theoretical uncertainties.

for example



can contribute to both
I=0 and I=2

+ transition for quark to hadron

If experimentally $\varepsilon' = O(\text{few} \times 10^{-7})$

$\rightarrow$ **it is likely that the standard model can explain CP violation**

$\rightarrow$ **it is unlikely that we can extract the parameter of the CKM matrix**

11) Three different mechanism of CP Violation

$$\eta_0 = \frac{A_0^{\mathrm{L}}}{A_0^{\mathrm{S}}}$$

$$= \frac{a_0 - (1-2\varepsilon_T)e^{-i\varphi_\Gamma} a_0^* e^{i(\varphi-\bar{\theta})}}{a_0 + (1-2\varepsilon_T)e^{-i\varphi_\Gamma} a_0^* e^{i(\varphi-\bar{\theta})}}$$

$$= \frac{1-(1-2\varepsilon_T)e^{i(\varphi-\bar{\theta}-2\varphi_{I=0}-\varphi_\Gamma)}}{1+(1-2\varepsilon_T)e^{i(\varphi-\bar{\theta}-2\varphi_{I=0}-\varphi_\Gamma)}}$$

$$\approx \varepsilon_{\mathrm{T}} - \frac{i}{2}(\varphi - \bar{\theta} - 2\varphi_{I=0} - \varphi_\Gamma)$$

$$\mathrm{Re}\,\eta = \mathrm{Re}\,\varepsilon_{\mathrm{T}}, \quad \mathrm{Im}\,\eta = \mathrm{Im}\,\varepsilon_{\mathrm{T}} - \frac{1}{2}(\varphi - \bar{\theta} - 2\varphi_{I=0} - \varphi_\Gamma)$$

$$\frac{A_{+-}}{\overline{A}_{+-}} = \frac{\sqrt{\frac{2}{3}}a_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}a_2 e^{i\delta_2}}{\left(\sqrt{\frac{2}{3}}a_0^* e^{i\delta_0} + \sqrt{\frac{1}{3}}a_2^* e^{i\delta_2}\right)e^{i(\varphi-\bar{\theta})}}$$

$$\approx (1 + 2\,\mathrm{Re}\,\varepsilon')e^{2i(\varphi_{I=0} + \mathrm{Im}\,\varepsilon' - \varphi - \bar{\theta})}$$

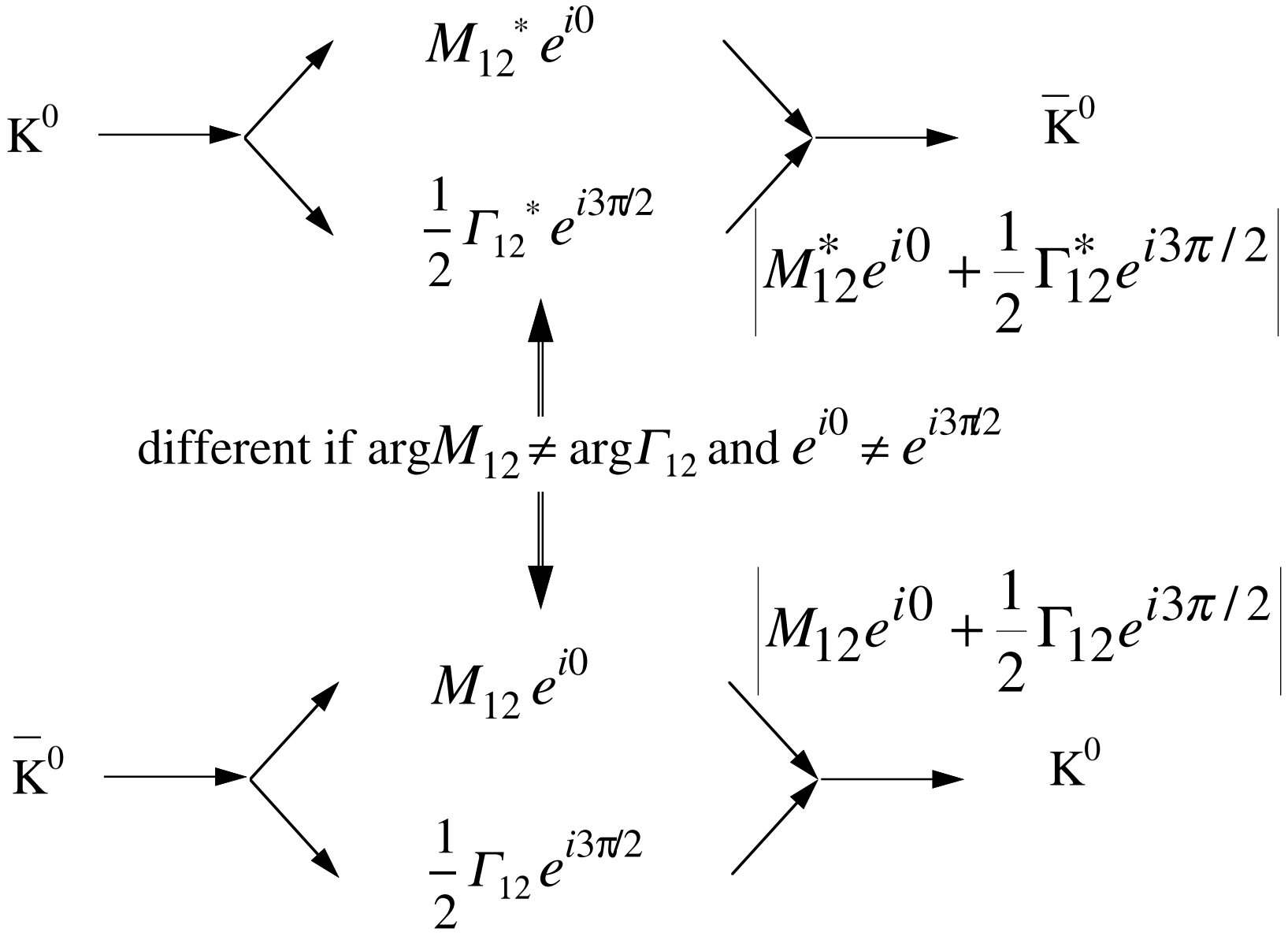
$$\varepsilon' = \frac{1}{\sqrt{2}}\,\mathrm{Im}\,\frac{a_2}{a_0}\,e^{i(\pi/2+\delta_2-\delta_0)}$$

$$\mathrm{Re}\,\varepsilon' = \frac{1}{\sqrt{2}}\,\mathrm{Im}\,\frac{a_2}{a_0}\sin(\delta_0 - \delta_2)$$

$\longrightarrow$

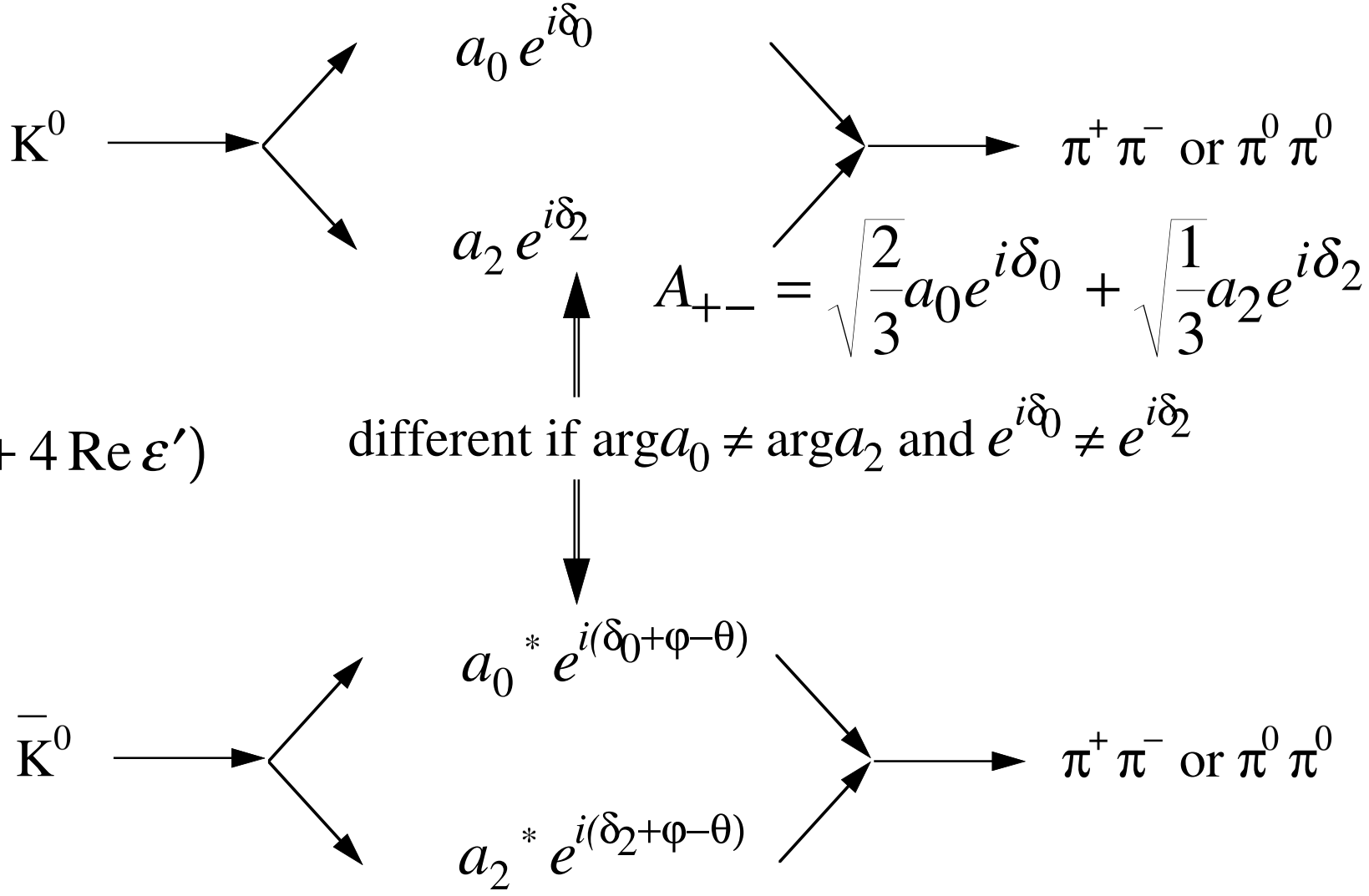
$\mathrm{Re}\,\varepsilon' = 0$
if
$\arg a_0 = \arg a_2$
OR
$\delta_0 = \delta_0$

a) CP violation in K^0-$\bar{K}^0$ oscillations = Re ε_T



Interference between M_{12} and Γ_{12}

b) CP violation in K^0-$\bar{K}^0$ decay amplitudes $= \mathrm{Re}\,\varepsilon'$

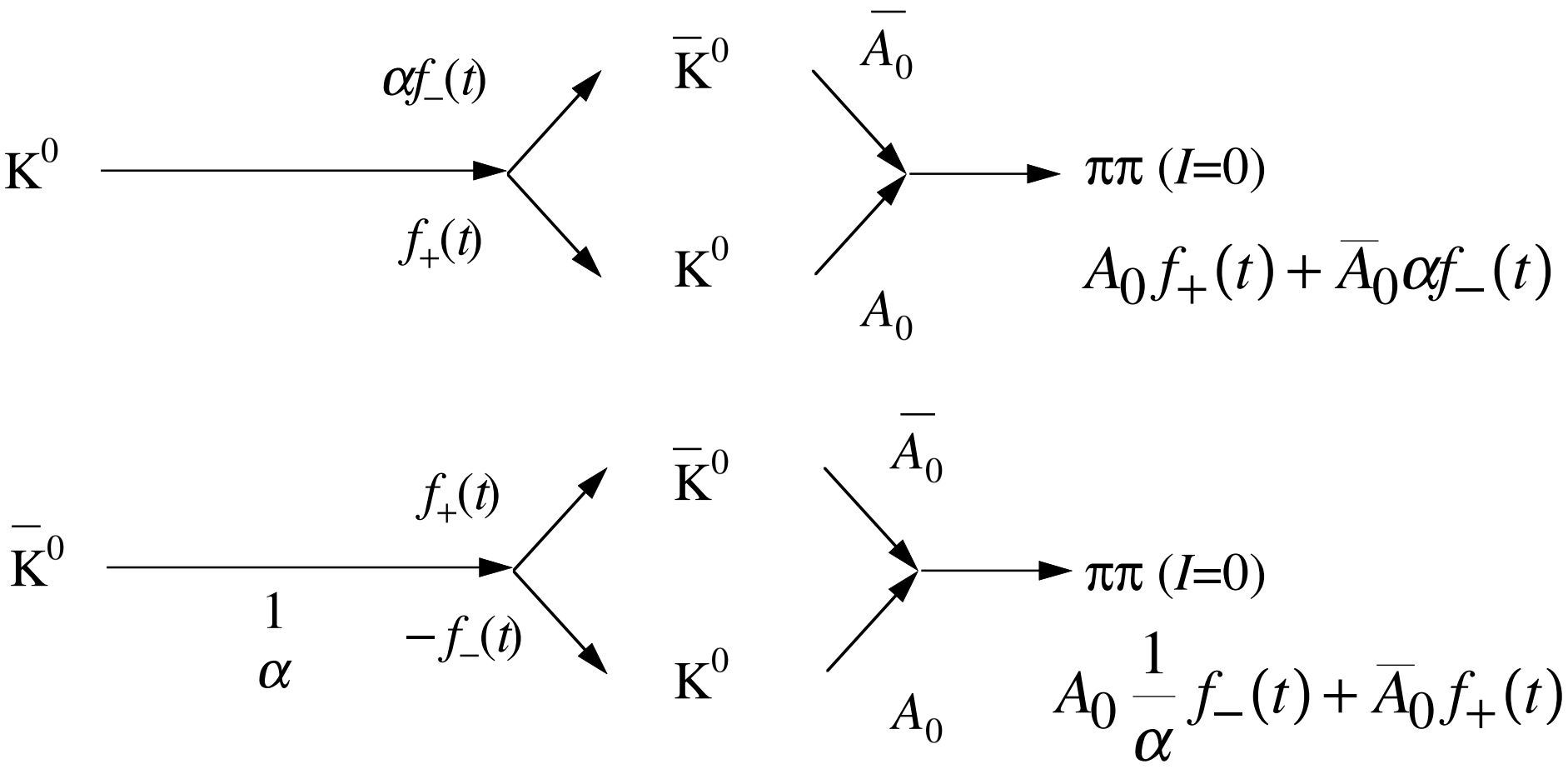


Interference between A_0 and A_2

$$\bar{A}_{+-} = \left(\sqrt{\frac{2}{3}} a_0^* e^{i\delta_0} + \sqrt{\frac{1}{3}} a_2^* e^{i\delta_2} \right) e^{i(\varphi-\bar{\theta})}$$

What about the imaginary part ?

c) CP violation in the interplay between K^0-$\bar{K}^0$ oscillations and decay= Im $\varepsilon_T - (\varphi - \bar{\theta} - 2\varphi_{I=0} - \varphi_\Gamma)/2$



$\mathrm{K}^0 \to 2\pi(I{=}0)$

$$f_+(t) + (1-2\varepsilon_{\mathrm{T}})e^{-i\varphi_\Gamma}\frac{\bar{A}_0}{A_0} f_-(t) \approx e^{i0} f_+(t) + e^{i(\varphi-\theta-2\varphi_{I=0}-\varphi_\Gamma)} f_-(t)$$

not oscillated

oscillated

invariant

invariant

complex conjugate

complex conjugate

$\bar{\mathrm{K}}^0 \to 2\pi(I{=}0)$

$$f_+(t) + (1+2\varepsilon_{\mathrm{T}})e^{i\varphi_\Gamma}\frac{A_0}{\bar{A}_0} f_-(t) \approx e^{i0} f_+(t) + e^{-i(\varphi-\theta-2\varphi_0-\varphi_\Gamma)} f_-(t)$$

If $\varphi - \bar{\theta} - 2\varphi_{I=0} - \varphi_\Gamma \neq 0$
there must be CP violation in the interference term

Difference in the "Rates" for

$$K^0_{t=0} \to 2\pi(I=0)\ (t)$$

and

$$\overline{K}^0_{t=0} \to 2\pi(I=0)\ (t)$$

are proportional to

$$\varphi - \theta - 2\varphi_{I=0} - \varphi_\Gamma$$

and

$$\mathrm{Im}\ \varepsilon_T$$

(For $\pi^+\pi^-$ final state, so as Im ε')

12) Experiments

Experiments based on K_L and K_S beam

Beams close to and far from targets NA31→NA48 Re(ε/ε′)

K_S from regenerated K_L E731→KTeV

Experiments with "almost" initially K^0 ($\bar{K}^0$) beam

Experiments with tagged K^0 and $\bar{K}^0$ beam

$K^{\pm}n \to K^0 p$, $K^- p \to K^0 n$

$pp \to K^0 K^- \pi^+$, $\bar{K}^0 K^+ \pi^-$ CPLEAR

Recent and near future ones

Experiments with coherent K^0-$\bar{K}^0$ pairs ϕ factories

global kaon studies

ε' measurements:

ratio of CP violating over CP conserving decay rates

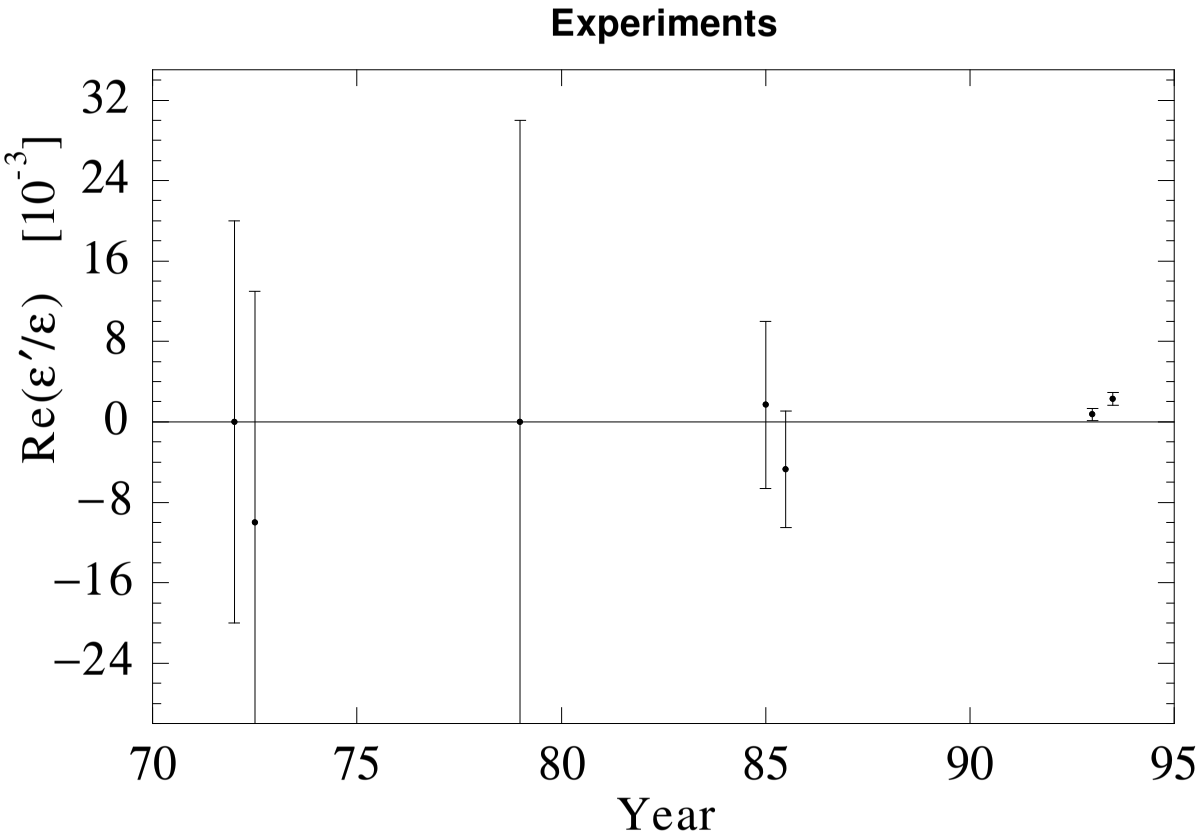
$$\frac{\dfrac{R\left(\mathrm{K_L} \to \pi^+\pi^-\right)}{R\left(\mathrm{K_S} \to \pi^+\pi^-\right)}}{\dfrac{R\left(\mathrm{K_L} \to \pi^0\pi^0\right)}{R\left(\mathrm{K_S} \to \pi^0\pi^0\right)}} = \frac{|\eta_{+-}|^2}{|\eta_{00}|^2}$$

$$= \frac{|\varepsilon_\mathrm{T} + \varepsilon'|^2}{|\varepsilon_\mathrm{T} - 2\varepsilon'|^2}$$

$$= 1 + 6\,\mathrm{Re}\,\frac{\varepsilon'}{\varepsilon_\mathrm{T}}$$

great help

impressive improvements have been made and still to come !!


Experiments
Re(ε′/ε) [10^-3]
32
24
16
8
0
−8
−16
−24
70
75
80
85
90
95
Year

13) $K\bar{K}$ system

$$\phi \quad \rightarrow \quad K\bar{K}$$

$s = 1$
$C = -1$
$P = -1$

$$\left| K\bar{K}_{L=1} \right\rangle = \int d\Omega_{\vec{e}_{p'}} \sum_{i=-1}^{1} Y_{1,m}\left(\vec{e}_{p'}\right) \left| K_{\vec{p}'} \right\rangle \left| \bar{K}_{-\vec{p}'} \right\rangle$$

$$\vec{e}_p = \vec{p} / |\vec{p}|$$

$L = 1$
$C = -1$
$P = -1$

note $Y_{l,m}(-\vec{e}_p) = (-1)^l Y_{l,m}(\vec{e}_p)$

Projection to a momentum state; $(\vec{p}, -\vec{p})$

$$\frac{1}{\sqrt{2}}\left[\left| K_{\vec{p}} \right\rangle \left| \bar{K}_{-\vec{p}} \right\rangle - \left| K_{-\vec{p}} \right\rangle \left| \bar{K}_{\vec{p}} \right\rangle \right]$$

For the neutral kaons, they oscillates !!

$$\left|\mathrm{K}^0\overline{\mathrm{K}}^0{}_{L=1}(t)\right\rangle = \int d\Omega_{\vec{e}_{p'}} \sum_{i=-1}^{1} Y_{1,m}\left(\vec{e}_{p'}\right)\left|\mathrm{K}^0{}_{\vec{p}'}(t)\right\rangle\left|\overline{\mathrm{K}}^0{}_{-\vec{p}'}(t)\right\rangle$$

$$= \int d\Omega_{\vec{e}_{p'}} \sum_{i=-1}^{1} Y_{1,m}\left(\vec{e}_{p'}\right)\left[f_+(t)\left|\mathrm{K}^0{}_{\vec{p}'}\right\rangle + \alpha f_-(t)\left|\mathrm{K}^0{}_{\vec{p}'}\right\rangle\right]$$

$$\times\left[f_+(t)\left|\overline{\mathrm{K}}^0{}_{-\vec{p}'}\right\rangle + \frac{1}{\alpha} f_+(t)\left|\overline{\mathrm{K}}^0{}_{-\vec{p}'}\right\rangle\right]$$

$$\left|\left\langle \mathrm{K}^0_{\vec{p}}\mathrm{K}^0_{-\vec{p}} \,\middle|\, \mathrm{K}^0\overline{\mathrm{K}}^0{}_{L=1}(t)\right\rangle\right|^2 = 0$$

$$\left|\left\langle \overline{\mathrm{K}}^0_{\vec{p}}\overline{\mathrm{K}}^0_{-\vec{p}} \,\middle|\, \mathrm{K}^0\overline{\mathrm{K}}^0{}_{L=1}(t)\right\rangle\right|^2 = 0$$

No $\overline{\mathrm{K}}^0$ $\overline{\mathrm{K}}^0$, K^0 K^0 states !!

$$\left|\left\langle \mathrm{K}^0_{\vec{p}}\overline{\mathrm{K}}^0_{-\vec{p}} \,\middle|\, \mathrm{K}^0\overline{\mathrm{K}}^0{}_{L=1}(t)\right\rangle\right|^2 \propto e^{-(\Gamma_\mathrm{L}+\Gamma_\mathrm{S})t}$$

System is always in the $\mathrm{K}^0\overline{\mathrm{K}}^0$ state

$$\left|\mathrm{K}^0\overline{\mathrm{K}}^0{}_{L=1}(t)\right\rangle = \int d\Omega_{\vec{e}_{p'}} \sum_{i=-1}^{1} Y_{1,m}\left(\vec{e}_{p'}\right)$$

$$\times e^{-im_{\mathrm{S}}t-\Gamma_{\mathrm{S}}t/2}\left|\mathrm{K}_{\mathrm{S}\vec{p}'}\right\rangle e^{-im_{\mathrm{L}}t-\Gamma_{\mathrm{L}}t/2}\left|\mathrm{K}_{\mathrm{L}-\vec{p}'}\right\rangle$$

No $\mathrm{K_L K_L}$ or $\mathrm{K_S K_S}$

It cannot decay into a same final state, e.g. $\pi^+\pi^-$ at the same time.

$$\left\langle \pi^+\pi^-{}_{\vec{p}}, \pi^+\pi^-{}_{-\vec{p}} \middle| H_{\mathrm{W}} \middle| \mathrm{K}^0\overline{\mathrm{K}}^0{}_{L=1}(t)\right\rangle = A_{\mathrm{S}}^{+-}{}_{\vec{p}} A_{\mathrm{S}}^{+-}{}_{-\vec{p}} \left(\eta_{+-} - \eta_{+-}\right) = 0$$

It can decay into $\pi^+\pi^-$ and $\pi^0\pi^0$ at the same time only if $\eta_{+-} \neq \eta_{00}$.

$$\left\langle \pi^+\pi^-{}_{\vec{p}}, \pi^0\pi^0{}_{-\vec{p}} \middle| H_{\mathrm{W}} \middle| \mathrm{K}^0\overline{\mathrm{K}}^0{}_{L=1}(t)\right\rangle = A_{\mathrm{S}}^{+-}{}_{\vec{p}} A_{\mathrm{S}}^{00}{}_{-\vec{p}} \left(\eta_{00} - \eta_{+-}\right)$$