Finding New Physics using heavy flavor decays

Fermilab Academic Lectures, May, 2014
Define Heavy Flavor Physics

- Flavor Physics: Study of interactions that differ among flavors: (quark flavors are u, d, c, s, b, t)
- Heavy: Not SM neutrino’s or u or d quarks, maybe s quarks, concentrate here on b quarks (some c), t too heavy
Physics Beyond the Standard Model

- **Baryogenesis**: From current measurements can only generate \( \frac{n_B - \bar{n}_B}{n_\gamma} \approx 10^{-20} \) but \( \approx 6 \times 10^{-10} \) is needed. Thus New Physics must exist to generate needed CP Violation.

- **Dark Matter**

- **Hierarchy Problem**: We don’t understand how we get from the Planck scale of Energy \( \sim 10^{19} \) GeV to the Electroweak Scale \( \sim 100 \) GeV without “fine tuning” quantum corrections.
Masses

12 orders of magnitude differences not explained; t quark as heavy as Tungsten
Formalism

- Standard model fermions

\[
\begin{pmatrix}
u \\ d
\end{pmatrix}_L \begin{pmatrix}
c \\ s
\end{pmatrix}_L \begin{pmatrix}
t \\ b
\end{pmatrix}_L, \quad u_R, \ d_R, \ c_R, \ s_R, \ t_R, \ b_R
\]

\[
\begin{pmatrix}
e^- \\ e_\nu
\end{pmatrix}_L \begin{pmatrix}
\mu^- \\ \nu_\mu
\end{pmatrix}_L \begin{pmatrix}
\tau^- \\ \nu_\tau
\end{pmatrix}_L, \quad e^-_R, \ \mu^-_R, \ \tau^-_R, \ \nu_{eR}, \ \nu_{\mu R}, \ \nu_{\tau R}.
\]

- SM gauge bosons: $\gamma, W^\pm, Z^0 & H^0$.

- Lagrangian for charged current interactions is

\[
L_{cc} = -\frac{g}{\sqrt{2}} J_{cc}^\mu W^\dagger_\mu + h.c.,
\]

- where

\[
J_{cc}^\mu = (\bar{\nu}_e, \ \bar{\nu}_\mu, \ \bar{\nu}_\tau) \gamma^\mu V_{MNS} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} + (\bar{u}_L, \ \bar{c}_L, \ \bar{t}_L) \gamma^\mu V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}
\]
Consider the charm quark. It forms a 2\textsuperscript{nd} generation doublet with the strange quark (c,s). Yet it also decays into the d quark which is in the first generation with the u quark (u,d).

We say this happens because the s & d quarks are “mixed” i.e. their wave functions really are described by a rotation matrix

\[
\begin{bmatrix}
    d' \\
    s'
\end{bmatrix} = \begin{bmatrix}
    \cos \theta_c & \sin \theta_c \\
    -\sin \theta_c & \cos \theta_c
\end{bmatrix} \begin{bmatrix}
    d \\
    s
\end{bmatrix} = \begin{bmatrix}
    V_{ud} & V_{us} \\
    V_{cd} & V_{cs}
\end{bmatrix} \begin{bmatrix}
    d \\
    s
\end{bmatrix}
\]

where the s' couples to c
Quark Mixing & CKM Matrix

- All 3 generations of -1/3 quarks (d, s, b) are mixed
- Described by CKM matrix (also $\nu$ are mixed)

$$V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
1 - \lambda^2 / 2 & \lambda & A\lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \lambda^2 / 2 - A\lambda^4 (1 + 4A^2) / 8 & A\lambda^2 \\
A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 + A\lambda^4 (1/2 + (\rho + i\eta)) & 1 - A^2 A^4 / 2
\end{pmatrix}$$

- Unitary 3x3 matrix can be described by 4 parameters $\lambda=0.225$, $A=0.8$, constraints on $\rho$ & $\eta$
- These are fundamental constants of nature in the Standard Model

Fermilab Academic Lectures, May, 2014
Why these values? Are the two related? Are they related to masses?
Y, formed of bb quarks, found at Fermilab in the \( \mu^+\mu^- \) channel

Followed by Doris Y, Y2; CLEO & CUSB that distinctly observed all 3 states, & published on the 1979 Xmas card
Discovery of Y(4S)

- The Y states were narrow, their observed widths were consistent with the experimental mass resolution, so below the threshold to decay into B\bar{B}
- Another resonance was found that was \(~20\) MeV wide, & subsequently shown to decay into either B^{+}B^{-} or B^{0}\bar{B}^{0}
B Experiments

- $e^+e^-$ at Y(4S) ARGUS, CLEO, BaBar, & Belle
- $e^+e^-$ at $Z^0$, LEP & SLC
- CDF & D0, 1.8 TeV $p\bar{p}$
- LHCb, CMS & ATLAS, 7-8 TeV $pp$
All detectors have cylindrical geometries with common elements

Key: PID, CsI ecal

Vertex detector usually Si strips, to measure B & B vertex separations, possible since beams in Belle & Babar have different energies; causes boost along beam direction. Typical resolutions on $\tau_B \sim 900$ fs.
Central detectors at pp

CDF

- Central Drift Chamber
- EM Calorimeter
- EM ShowerMax
- Hadron Calorimeter
- Muon Detector
- Steel (Magn. yokes)

ISL Si Layers
SYX-II Si detector
Solenoidal Magnet
The LHC

- 4 TeV x 4 TeV pp collisions (future ~7 x ~7)
Overall view of the LHC experiments.

27 km in circumference
Detector Geometry

- Complementary to ATLAS & CMS
- Much less expensive
The primary pp collision produces a pair of $b\bar{b}$ quarks. They then form hadrons. In the forward region at LHC the $b\bar{b}$ production $\sigma$ is large.

The hadrons containing the $b$ & $\bar{b}$ quarks are both likely to be in the acceptance. Essential for knowing if a neutral B meson started out as a $B^0$ or $\bar{B}^0$, determined by “flavor tagging”

At $\mathcal{L} = 2 \times 10^{32}$/cm$^2$-s, we get $\sim 10^{12}$ B hadrons in $10^7$ sec

Fermilab Academic Lectures, May, 2014
Detector Workings

LHCb detector ~ fully installed and commissioned  → walk through the detector using the example of a $B_s \rightarrow D_s K$ decay

Fermilab Academic Lectures, May, 2014
**B-Vertex Measurement**

**Example:** $B_s \rightarrow D_s K$

- 47 $\mu$m
- 144 $\mu$m
- $K^+$
- $K^-$
- $\pi^-$

$D_s$ Primary vertex

$d \sim 1$ cm

440 $\mu$m

Decay time resolution = 40 fs

**Vertex Locator** (Velo)
Silicon strip detector with
$\sim 5$ $\mu$m hit resolution
$\rightarrow 30$ $\mu$m IP resolution

Vertexing:
- trigger on impact parameter
- measurement of decay distance & decay time $= d/v = md/p$

Double Gaussian fit
$\sigma_1 = 33 \pm 1$ fs
$\sigma_2 = 67 \pm 3$ fs (31%)

$\sigma(\tau) \sim 40$ fs
Momentum and Mass measurement

Momentum meas. + direction (VELO):
Mass resolution for background suppression

$\text{B}_s \rightarrow D_s^- K^+$

Mass resolution $\sigma \sim 15\text{ MeV}$
Hadron Identification

**RICH: K/π identification using Cherenkov light emission angle**

**RICH1**: 5 cm aerogel $n=1.03$

$4 \text{ m}^3 \text{C}_4\text{F}_{10} n=1.0014$

**RICH2**: 100 m$^3$ CF$_4$ $n=1.0005$

**Fermilab Academic Lectures, May, 2014**
**Calorimetry and L0 trigger**

**Calorimeter system:**
- Identify electrons, hadrons, $\pi^0$, $\gamma$
- Level 0 trigger: high $E_T$ electron and hadron

**ECAL (inner modules):** $\sigma(E)/E \sim 8.2\%/\sqrt{E} + 0.9\%$

---

**Fermilab Academic Lectures, May, 2014**
Muon system:
• Level 0 trigger: High $P_t$ muons
• OS flavour tagging
Triggering

Trigger is crucial as $\sigma_{bb}$ is less than 1% of total inelastic cross section and $B$ decays of interest typically have $B$ branching ratios of $<10^{-5}$

Hardware level (L0)
- Search for high-$p_T$ $\mu$, $e$, $\gamma$ and hadron candidates

Software level (High Level Trigger, HLT)
- Farm with $\mathcal{O}(29000)$ multi-core processors
- Very flexible algorithms, writes $\sim$5 kHz to storage

This is the bottleneck
Detector Performance

- Detector works better than expected
- Run at $4 \times 10^{-32}$ cm$^{-2}$/s instead of $2 \times 10^{32}$, with fewer bunches in the machine which is more difficult $\sim<1.5>$ interactions/crossing
- Detector efficiency $>95\%$ for all systems
- Problems: Vertex resolution slightly worse, flavor tagging somewhat poorer
- Luminosity is leveled – small changes of L with time; beams are brought closer together when currents decrease
Luminosity Leveling

- Luminosity is maintained as at a constant value of $\sim 4 \times 10^{32}/\text{cm} \cdot \text{s}$ by displacing beams transversely.
- Integral $L$ is $1/\text{fb}$ in 2011, collected $2/\text{fb}$ more in 2012.
$B^- \rightarrow J/\psi \ K^-$

LHCb Event Display
20 MHz of bunch crossing (in 2012, with 50 ns bunch spacing) with an average of 1.5 pp interactions per bunch crossing → this level of pileup not an issue for LHCb
Consider the b decay of $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$.

The decay width is given by

$$\Gamma_\mu = \frac{G_F^2}{192\pi^3} m_\mu^5 \times \text{(phase space)} \times \text{(radiative corrections)}$$

Since $\Gamma_\mu \otimes \tau_\mu = \hbar$, measuring the muon lifetime determines $G_F$. 

Fermilab Academic Lectures, May, 2014
$|V_{us}| = 0.97418 \pm 0.00026$

is measured using nuclear $\beta$ decays

For $|V_{us}|$ use semileptonic kaon decays. The decay width is given by

$$\Gamma(K_{l3}) = \frac{C_K^2 G_F^2 M_K^5}{192 \pi^3} S_{EW} |V_{us}|^2 |f_+(0)|^2 \times I_{K,l}(\lambda) \left(1 + 2 \Delta_{SU}^{(2)} K + 2 \Delta_{EM}^{(2)} K, l \right)$$

- $C_K$ is a Clebsch-Gordan coefficient = $1/2$
- $S_{EW}$ is the short-distance EW correction = $1.0232$
- $\Delta$’s are SU(2) breaking & long-distance E&M corrects
- $I_{K,l}(\lambda)$ is the phase space integral
\[ f_+(0) \text{: Here we have quark transition, yet the quarks have to form a single hadron, the } \pi^0 \]

- The probability of this happening is parameterized in terms of the 4-momentum transfer squared, \( q^2 = (p-p')^2 \). From the fact that the \( K \to \pi \) weak transition must be Vector

\[
\pi(p') V_\mu = \gamma_\mu (1+\gamma_5) K(p) = (p_\mu + p'_\mu) f_+(q^2) + (p_\mu - p'_\mu) f_-(q^2)
\]

- For massless leptons the \( f_-(q^2) \) term vanishes

- The shape of \( f(q^2) \) can be measured, so only \( f_+(0) \) remains to be calculated.
Measurements of $f_+(0)|V_{us}|$

- $f_+(0) = 0.964(5)$
- $\lambda = |V_{us}| = 0.2246 \pm 0.0012$

Experiment measures $K$ lifetime, shape of form-factor & value of the form-factor at $q^2 = 0$
Basic decay diagram:

- Two methods used to determine $|V_{cb}|$ from data: **Exclusive**, only a $D$ or $D^*$ produced, & **Inclusive**, take all $b \rightarrow c$ decays

- If $B \rightarrow D$ one form-factor, for $B \rightarrow D^*$, have 3
Based on HQET invented by N. Isgur & M. Wise

- Idea is that there are spin & flavor symmetries between two $\infty$ heavy quarks; the b & c quarks are not quite that heavy, but corrections can be calculated in a controlled way. In HQET only 1 ff for $B \rightarrow D^*$, where there are 3 independent spin states.

- Consider the invariant 4-velocity transfer, $\omega$. When $\omega=1$, the b transforms into a c with the same velocity, so the form-factor is unity modulo some small corrections.

- Note $\omega = \left( \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_Bm_{D^*}} \right)$
Exclusive $|V_{cb}|$

- $F(\omega)$ is the form-factor

$$\frac{d\Gamma(B \rightarrow D^* l\nu)}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} K(\omega) F(\omega)^2$$

- $K(\omega)$ is the phase space factor, which goes to zero as $\omega \rightarrow$, so data must be extrapolated. There are theoretical models for the shape of $F(\omega)$. All that’s necessary is the lifetime, the value of the branching fraction at $F(1)$, which determines $(F(1)|V_{cb}|)^2$, & the theoretically determined corrections to $F(1)$ from 1
Predictions of $F(1)$

- **Lattice (FNAL/MILC):**
  - $0.906 \pm 0.004 \pm 0.012$
- **QCD sum rules**
  - $0.86 \pm 0.02$

$|V_{cb}| \times 10^3 = 39.04 \pm 0.49_{\text{exp}} \pm 0.53_{\text{QCD}} \pm 0.19_{\text{QED}} \text{ (Lattice)}$

$= 41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{thy}} \text{ (Sum rules)}$
Here assume that the ensemble of exclusive $b \rightarrow c$ decays, $B \rightarrow D l \nu$, $D^* l \nu$, $D^{**} l \nu$,… can be approximated by a continuum, called “duality”. The model is called the Heavy Quark Expansion (HQE).

The decay rate is related to $|V_{cb}|$ as

$$\Gamma(B \rightarrow X_c \ell \bar{\nu}) = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3} \left( f(\rho) + k(\rho) \frac{\mu^2_\pi}{2m_b^2} + g(\rho) \frac{\mu^2_G}{2m_b^2} \right)$$

$$+ d(\rho) \frac{\rho^3_D}{m_b^3} + l(\rho) \frac{\rho^3_{LS}}{m_b^3} + \mathcal{O}(m_b^{-4})$$

where $\rho = m_c^2/m_b^2$, and $\mu^2_\pi$, $\mu^2_G$, $\rho_D$ and $\rho_{LS}$ are non-perturbative matrix elements of local operators.

We will not go into the details here see arXiv:0902.3743
Inclusive $|V_{cb}|$ II

- Latest result: $|V_{cb}| \times 10^3 = 41.94 \pm 0.43_{\text{fit}} \pm 0.59_{\text{thy}}$
  
  $= 41.94 \pm 0.73$

- Exclusive (Lattice) $= 39.04 \pm 0.75$

- Difference has $\chi^2 = 3.8$ for 1 dof, prob = 5%

- Could there be a problem here?

- $\Lambda_b/B^0$ lifetime ratio: HQE predicts that the lifetime ratio is almost equal, with $\Lambda_b$ being shorter by a few %.
$\Lambda_b / B^0$ lifetime ratio

- $\Lambda_b$ lifetime measurements were much lower
- LHCb now finds

$$\frac{\tau_{\Lambda_b^0}}{\tau_{B^0}} = 0.974 \pm 0.006 \pm 0.004.$$ 

- Consistent with HQE original prediction.

Credit Uraltsev

Fermilab Academic Lectures, May, 2014
**Exclusive $|V_{ub}|$**

- No theory like HQET
- Must rely on Lattice & model calculations

---

### Exclusive decays

<table>
<thead>
<tr>
<th>Process</th>
<th>Theory/Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow \pi l \bar{\nu}_l$</td>
<td>HPQCD ($q^2 &gt; 16$) (HFAG)&lt;sup&gt;97,11&lt;/sup&gt;</td>
<td>$3.52 \pm 0.08^{0.61}_{0.40}$</td>
</tr>
<tr>
<td></td>
<td>Fermilab/MILC ($q^2 &gt; 16$) (HFAG)&lt;sup&gt;98,11&lt;/sup&gt;</td>
<td>$3.36 \pm 0.08^{0.37}_{0.31}$</td>
</tr>
<tr>
<td></td>
<td>lattice, full $q^2$ range (HFAG)&lt;sup&gt;11&lt;/sup&gt;</td>
<td>$3.28 \pm 0.29$</td>
</tr>
<tr>
<td></td>
<td>LCSR ($q^2 &lt; 12$) (HFAG)&lt;sup&gt;100,11&lt;/sup&gt;</td>
<td>$3.41 \pm 0.06^{0.37}_{-0.32}$</td>
</tr>
<tr>
<td></td>
<td>LCSR ($q^2 &lt; 16$) (HFAG)&lt;sup&gt;101,11&lt;/sup&gt;</td>
<td>$3.58 \pm 0.06^{0.59}_{-0.40}$</td>
</tr>
</tbody>
</table>

---

See Ricciardi arXiv:1403.7750

---

Fermilab Academic Lectures, May, 2014
## Exclusive $|V_{ub}|$

- Use HQE. Here many final states possible

<table>
<thead>
<tr>
<th>Models</th>
<th>BNLP</th>
<th>GGOU</th>
<th>ADFR</th>
<th>DGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Babar</td>
<td>4.28 ± 0.24$^{+0.18}_{-0.20}$</td>
<td>4.35 ± 0.24$^{+0.09}_{-0.10}$</td>
<td>4.29 ± 0.24$^{+0.18}_{-0.19}$</td>
<td>4.40 ± 0.24$^{+0.12}_{-0.13}$</td>
</tr>
<tr>
<td>Belle</td>
<td>4.47 ± 0.27$^{+0.19}_{-0.21}$</td>
<td>4.54 ± 0.27$^{+0.10}_{-0.11}$</td>
<td>4.48 ± 0.30$^{+0.19}_{-0.19}$</td>
<td>4.60 ± 0.27$^{+0.11}_{-0.13}$</td>
</tr>
<tr>
<td>HFAG</td>
<td>4.40 ± 0.15$^{+0.19}_{-0.21}$</td>
<td>4.39 ± 0.15$^{+0.12}_{-0.20}$</td>
<td>4.03 ± 0.13$^{+0.18}_{-0.12}$</td>
<td>4.45 ± 0.15$^{+0.18}_{-0.16}$</td>
</tr>
</tbody>
</table>

- So take e.g. exclusive $(3.28±0.29)x10^{-3}$
- & inclusive $(4.20 ±0.25)x10^{-3}$
- These are inconsistent!
- No resolution in sight

See Ricciardi arXiv:1403.7750
Summary

Note

\[ \bar{\rho} = \rho (1 - \lambda^2 / 2) \]

\[ \bar{\eta} = \eta (1 - \lambda^2 / 2) \]

Bands are ±2σ
Neutral Meson Mixing

- Neutral heavy mesons can transform into their anti-particles via 2nd order weak interactions.
- Short distance transition rate depends on:
  - mass of intermediate $q_i$, the heavier the larger, favors mesons containing s & b, since t is allowed
  - CKM elements $V_{ij}$.

New particles possible in the loop

Fermilab Academic Lectures, May 2014
Mixing formalism

- Hamiltonian

\[ \mathcal{H} = M - \frac{i}{2} \Gamma = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix} \]

- Schrödinger equation

\[ i \frac{d}{dt} \begin{pmatrix} |B^0(t)\rangle \\ \overline{B^0(t)} \end{pmatrix} = \mathcal{H} \begin{pmatrix} |B^0(t)\rangle \\ \overline{B^0(t)} \end{pmatrix} \]

- Diagonalizing

\[ \Delta m = m_{B_H} - m_{B_L} = 2 |M_{12}| \]

\[ \Delta \Gamma = \Gamma_L - \Gamma_H = 2 |\Gamma_{12}| \cos \phi \]

\[ \phi = \arg \left( - \frac{M_{12}}{\Gamma_{12}} \right) \]
B Mixing data

First seen by ARGUS

\[ \Delta m_d = 0.5156 \pm 0.0051 \, \text{(stat)} \pm 0.0033 \, \text{(syst)} \, \text{ps}^{-1} \]

Fermilab Academic Lectures, May, 2014
D⁰-⁻D⁰ Mixing

- D*+ → π⁺D⁰ provides an initial flavor tag
- “Wrong-sign” (WS) D⁰ can appear via mixing or a rare decay that gives the same final state called doubly-Cabbibo suppressed decay (DCS), where DCS follow \( \sim \exp(-t/\tau_{D⁰}) \). Mixing, however, depends on \( t \) in a more complicated way.
- Define \( R_D = \text{DCS}/(\text{Cabibbo favored}) \). Mixing is parameterized as \( x' \) & \( y' \), functions of \( \Delta m \) & \( \Delta \Gamma \).
- Measure Wrong-sign/Right-sign, \( R(t) = (\text{WS}/\text{RS}) \)

\[
R(t) \approx R_D + \sqrt{R_D} \cdot y' \cdot \frac{t}{\tau} + \frac{x'^2 + y'^2}{4} \left( \frac{t}{\tau} \right)^2
\]
Charm mixing result

\[ D^{*+} \rightarrow \pi^+ D^0, \]
\[ D^0 \rightarrow K^- \pi^+ \text{ (RS)} \]
\[ \bar{D}^0 \rightarrow K^+ \pi^- \text{ (WS)} \]

No mixing excluded at 9.1σ, systematic errors are included:
\[ y' = (7.2 \pm 2.4)\% \]
\[ x'^2 = (-0.09 \pm 0.13)\% \]

Fermilab Academic Lectures, May, 2014
For $B^0$ mixing

$$\frac{\Delta m}{\Gamma} = \frac{G_F^2}{6\pi^2} B_{B_d} f_B^2 m_B \tau_B |V_{tb}^* V_{td}|^2 m_t^2 F \left( \frac{m_t^2}{M_W^2} \right) \eta_{QCD}.$$ 

$B_B$ is a theoretical parameter, $f_B$, the meson decay constant is also estimated theoretically though in principle measuring $B^- \rightarrow \tau \nu$ would determine $|V_{ub}|^2 f_B^2$. $F$ is a known function & $\eta_{QCD} \sim 0.8$

Similar Eq. for $B_s$ mixing. Errors cancel in

$$\frac{x_d}{x_s} = \frac{B_B}{B_{B_s}} \frac{f_B^2}{f_{B_s}^2} \frac{m_B}{m_{B_s}} \frac{\tau_B}{\tau_{B_s}} \frac{|V_{tb}^* V_{td}|^2}{|V_{tb}^* V_{ts}|^2}.$$
B mixing & CKM constraints

- We have

\[ |V_{tb}^* V_{td}|^2 = A\lambda^3 |(1 - \rho - i\eta)|^2 = A\lambda^3 (\rho - 1)^2 + \eta^2 \] and

\[ |V_{tb}^* V_{ts}|^2 = A\lambda^2 , \]

- So the ratio gives a circle in the \((\bar{\rho}, \bar{\eta})\) plane centered at \((1,0)\).

- (Modulo small higher order corrections)
Sakharov conditions

- Big bang gave matter & anti-matter
- For the Universe to exist:
  1. **Baryon # violation**
  2. **Departure from thermal equilibrium**
  3. **C & CP violation**, where C is charge conjugation, e.g., $C|p> = ±|p>$, & P is parity $P|\psi(r)> = ±|\psi(-r)>$
    - 1. is satisfied as SM gives B violation at high T
    - 2. is satisfied from the EW phase transition
    - 3. C & CP are violated by weak interactions
- **BUT** amount of CPV is too small by $10^9$, so new sources need to be found

Fermilab Academic Lectures, May, 2014
CP formalism

- Basic idea: two interfering amplitudes that ultimately involve the CKM parameter $\eta$.

\[ \Gamma(B \to f) = \left( |A| e^{i(s_A + w_A)} + |B| e^{i(s_B + w_B)} \right)^2 \]
\[ \Gamma(\bar{B} \to \bar{f}) = \left( |A| e^{i(s_A - w_A)} + |B| e^{i(s_B - w_B)} \right)^2 \]
\[ \Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f}) = 2 |AB| \sin(s_A - s_B) \sin(w_A - w_B) \]

- Favorable if $A$ & $B$ are about the same size
- Resulting rate difference depends on both a strong & weak phase difference
Consider specifically $|B^0>$, but this can be for any $P^0$: $K^0$, $B^0$, $B^0_s$, or $D^0$.

$CP|B^0> = |\bar{B}^0>$. So these are not CP eigenstates, but

$|B_1^0> = \frac{1}{\sqrt{2}} (|B^0> - |\bar{B}^0>)$ \& $|B_2^0> = \frac{1}{\sqrt{2}} (|B^0> + |\bar{B}^0>)$ are with $CP|B_1^0> = |B_1^0>$ \& $CP|B_2^0> = -|B_2^0>$

To allow for CPV define

$|B_L> = p|B^0> + q|\bar{B}^0>$, $|B_H> = p|B^0> - q|\bar{B}^0>$

where CP is violated if $|p/q| \neq 1$
Here we are interested in a final state that can be reached by either a $|P^0>$ or a $|\overline{P}^0>$

Then we can utilize mixing to provide another Interfering amplitude

$f$ can be a CP eigenstate, $CP|f_{CP}\rangle = \pm |f_{CP}\rangle$

Define $A = \langle f_{CP}|H|B^0\rangle$, $\overline{A} = \langle f_{CP}|H|\overline{B}^0\rangle$. If $|\frac{\overline{A}}{A}| \neq 1$ we have “direct” CPV, but all that is needed is for $\lambda = \frac{q}{p} \cdot \frac{\overline{A}}{A} \neq 1$ which can happen even if $|\frac{p}{q} = \frac{\overline{A}}{A}| = 1$
The asymmetry is given by

$$ a_{f_{CP}} = \frac{\Gamma(B^0(t) \rightarrow f_{CP}) - \Gamma(\overline{B}^0(t) \rightarrow f_{CP})}{\Gamma(B^0(t) \rightarrow f_{CP}) + \Gamma(\overline{B}^0(t) \rightarrow f_{CP})} $$

$$ a_{f_{CP}} = \frac{(1 - |\lambda|^2) \cos(\Delta mt) - 2\text{Im}\lambda \sin(\Delta mt)}{1 + |\lambda|^2} $$

For $|\lambda|=1$, we have

$$ a_{f_{CP}} = -\text{Im}\lambda \sin(\Delta mt) $$
CP mixing phase

- Depends on CKM elements in mixing or box diagram

For $B^0$

$$q = \frac{(V_{tb}^* V_{td})^2}{|V_{tb} V_{td}^*|^2} = \frac{(1-\rho-i\eta)^2}{(1-\rho+i\eta)(1-\rho-i\eta)} = e^{-2i\beta}$$

arg($p/q$)=$\beta$

For $B_s$

$$q = \frac{(V_{tb}^* V_{ts})^2}{|V_{tb} V_{ts}^*|^2} = 1$$

arg($p/q$)$\sim$0

Fermilab Academic Lectures, May, 2014
Need $p/q$ and $\bar{A}/A$. Choosing a suitable CP eigenstate forces $\bar{A}/A=1$. $p/q$ comes from mixing:

$$\frac{q}{p} = \frac{(V_{tb}^* V_{td})^2}{|V_{tb} V_{td}|^2} = \frac{(1 - \rho - i\eta)^2}{(1 - \rho + i\eta)(1 - \rho - i\eta)} = e^{-2i\beta}$$

$B^0$:

$$\text{Im} \left( \frac{q}{p} \right) = -\frac{2(1 - \rho)\eta}{(1 - \rho)^2 + \eta^2} = \sin(2\beta)$$

This is SM
$B^0 \rightarrow \{c\bar{c}\} K^0$

For charmonium final states (Belle)
Measurements of $\sin^2 \beta$

- Requires knowledge of B flavor at birth – use info from the other B in the event
- $\sin^2 \beta$ values
  - Belle: $0.667 \pm 0.023 \pm 0.012$
  - BaBar: $0.691 \pm 0.028 \pm 0.012$
- World Average: $0.682 \pm 0.019$

\[ \beta = (21.5^{+0.8}_{-0.7})^0 \text{ or } (68.5^{+0.7}_{-0.8})^0 \]
For $f = J/\psi \phi$ or $J/\psi f_0$

$\mathbb{B}_s^0 \left\{ \begin{array}{c} b \\ \bar{s} \end{array} \right\} \rightarrow W^- \left\{ \begin{array}{c} c \\ \bar{c} \end{array} \right\} J/\psi$

Small CPV expected, good place for NP to appear. Non zero due to CKM effects of order $\lambda^4$ in $V_{ts}$

$J/\psi \phi$ not a CP eigenstate. Why? But can be used

$$\phi_s^{SM} = -2 \beta_s = -2 \arg \left( -\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right) = -2^\circ$$
Consider
\[ a[f(t)] = \frac{\Gamma(\bar{M} \rightarrow f) - \Gamma(M \rightarrow f)}{\Gamma(\bar{M} \rightarrow f) + \Gamma(M \rightarrow f)} \]

Define
\[ A_f \equiv A(M \rightarrow f), \quad \bar{A}_f \equiv A(\bar{M} \rightarrow f), \quad \lambda_f = \frac{p \bar{A}_f}{q A_f} \]

Only 1 \( A_f \) & \( \Delta \Gamma = 0 \)
\[ \Gamma(M \rightarrow f) = N_f |A_f|^2 e^{-\Gamma t} \left( 1 - \text{Im} \lambda_f \sin(\Delta M t) \right) \]

Then
\[ a[f(t)] = -\text{Im} \lambda_f, \text{ & } \lambda_f \text{ is a function of } V_{ij} \text{ in SM} \]

For \( B^0, \Delta \Gamma H0 \), but there can be multiple \( A_f \)
\[ \Gamma(M \rightarrow f) = N_f |A_f|^2 e^{-\Gamma t} \left( \frac{1 - |\lambda_f|^2}{2} \cos(\Delta M t) - \text{Im} \lambda_f \sin(\Delta M t) \right) \]

If in addition \( \Delta \Gamma \neq 0 \), eg. \( B_s \)
\[ \Gamma(M \rightarrow f) = N_f |A_f|^2 e^{-\Gamma t} \left( \frac{1 + |\lambda_f|^2}{2} \cosh \frac{\Delta M t}{2} + \frac{1 - |\lambda_f|^2}{2} \cos(\Delta M t) - \text{Re} \lambda_f \sinh \frac{\Delta M t}{2} - \text{Im} \lambda_f \sin(\Delta M t) \right) \]

See Nierste
Transversity

\[
\frac{d^4 \Gamma(B_s^0 \rightarrow J/\psi \phi)}{dt \, d \cos \theta \, d \varphi \, d \cos \psi} \equiv \frac{d^4 \Gamma}{dt \, d \Omega} \propto \sum_{k=1}^{10} h_k(t) f_k(\Omega)
\]

<table>
<thead>
<tr>
<th>(k)</th>
<th>(h_k(t))</th>
<th>(f_k(\theta, \psi, \varphi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(</td>
<td>A_0</td>
</tr>
<tr>
<td>2</td>
<td>(</td>
<td>A_\parallel(t)</td>
</tr>
<tr>
<td>3</td>
<td>(</td>
<td>A_\perp(t)</td>
</tr>
<tr>
<td>4</td>
<td>(\Im(A_\parallel(t) A_\perp(t)))</td>
<td>(-\sin^2 \psi \sin 2\theta \sin \phi)</td>
</tr>
<tr>
<td>5</td>
<td>(\Re(A_0(t) A_\parallel(t)))</td>
<td>(\frac{1}{2} \sqrt{2} \sin 2\psi \sin^2 \theta \sin 2\phi)</td>
</tr>
<tr>
<td>6</td>
<td>(\Im(A_0(t) A_\perp(t)))</td>
<td>(\frac{1}{2} \sqrt{2} \sin 2\psi \sin 2\theta \cos \phi)</td>
</tr>
<tr>
<td>7</td>
<td>(</td>
<td>A_s(t)</td>
</tr>
<tr>
<td>8</td>
<td>(\Re(A_s^*(t) A_\parallel(t)))</td>
<td>(\frac{1}{3} \sqrt{6} \sin \psi \sin^2 \theta \sin 2\phi)</td>
</tr>
<tr>
<td>9</td>
<td>(\Im(A_s^*(t) A_\perp(t)))</td>
<td>(\frac{1}{3} \sqrt{6} \sin \psi \sin 2\theta \cos \phi)</td>
</tr>
<tr>
<td>10</td>
<td>(\Re(A_s^*(t) A_0(t)))</td>
<td>(\frac{4}{3} \sqrt{3} \cos \psi \left(1 - \sin^2 \theta \cos^2 \phi\right))</td>
</tr>
</tbody>
</table>

For S-wave under \(\phi\) predicted by Stone & Zhang PRD 79, 074024 (2009)
Transversity II

\[
|A_0|^2(t) = |A_0|^2 e^{-\Gamma_s t} [\cosh \left( \frac{\Delta \Gamma}{2} t \right) - \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) + \sin \phi_s \sin(\Delta m t)],
\]

\[
|A_\parallel|^2(t) = |A_\parallel|^2 e^{-\Gamma_s t} [\cosh \left( \frac{\Delta \Gamma}{2} t \right) - \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) + \sin \phi_s \sin(\Delta m t)],
\]

\[
|A_\perp|^2(t) = |A_\perp|^2 e^{-\Gamma_s t} [\cosh \left( \frac{\Delta \Gamma}{2} t \right) + \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \sin \phi_s \sin(\Delta m t)],
\]

\[
\Im(A_\parallel^*(t) A_\perp(t)) = |A_\parallel| |A_\perp| e^{-\Gamma_s t} [-\cos(\delta_\perp - \delta_\parallel) \sin \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \cos(\delta_\perp - \delta_\parallel) \cos \phi_s \sin(\Delta m t) + \sin(\delta_\perp - \delta_\parallel) \cos(\Delta m t)],
\]

\[
\Re(A_0^*(t) A_\parallel(t)) = |A_0| |A_\parallel| e^{-\Gamma_s t} \cos(\delta_\parallel - \delta_0) [\cosh \left( \frac{\Delta \Gamma}{2} t \right) - \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) + \sin \phi_s \sin(\Delta m t)],
\]

\[
\Im(A_0^*(t) A_\perp(t)) = |A_0| |A_\perp| e^{-\Gamma_s t} [-\cos(\delta_\perp - \delta_0) \sin \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \cos(\delta_\perp - \delta_0) \cos \phi_s \sin(\Delta m t) + \sin(\delta_\perp - \delta_0) \cos(\Delta m t)],
\]

\[
|A_s|^2(t) = |A_s|^2 e^{-\Gamma_s t} [\cosh \left( \frac{\Delta \Gamma}{2} t \right) + \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \sin \phi_s \sin(\Delta m t)],
\]

\[
\Re(A_s^*(t) A_\parallel(t)) = |A_s| |A_\parallel| e^{-\Gamma_s t} [-\sin(\delta_\parallel - \delta_s) \sin \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \sin(\delta_\parallel - \delta_s) \cos \phi_s \sin(\Delta m t) + \cos(\delta_\parallel - \delta_s) \cos(\Delta m t)],
\]

\[
\Im(A_s^*(t) A_\perp(t)) = |A_s| |A_\perp| e^{-\Gamma_s t} [\sin(\delta_\perp - \delta_s) [\cosh \left( \frac{\Delta \Gamma}{2} t \right) + \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \sin \phi_s \sin(\Delta m t)],
\]

\[
\Re(A_s^*(t) A_0(t)) = |A_s| |A_0| e^{-\Gamma_s t} [-\sin(\delta_0 - \delta_s) \sin \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \sin(\delta_0 - \delta_s) \cos \phi_s \sin(\Delta m t) + \cos(\delta_0 - \delta_s) \cos(\Delta m t)].
\]

*only term for \(f=f_{cp}\)*
Reconstructed π⁺π⁻ mass spectrum

In region between arrows, measured to be >97.7%

CP-odd @95% cl

\[ \alpha[f(t)] \equiv 2 \sin \phi_s \sin(\Delta M t) \]

\[ \phi_s = -0.019^{+0.173+0.004}_{-0.174-0.003} \text{ rad (1/fb)} \]

(uncertainty for 3/fb~0.070 rad)

Fermilab Academic Lectures, May, 2014
Combining LHCb results:

\[ \phi_s = 0.001 \pm 0.101 \pm 0.027 \text{ (rad)} \]

\[ \Gamma = 0.6580 \pm 0.0054 \pm 0.0066 \text{ (ps}^{-1}) \]

\[ \Delta \Gamma = 0.116 \pm 0.018 \pm 0.006 \text{ (ps}^{-1}) \]