Time Reversal Violation: Suggested Tests for Non-Leptonic Two-Body Decays

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Abstract

We discuss a theoretical condition for time reversal violation in weak non-leptonic two-body decays involving spinning and unstable particles in the final state. In some cases this condition looks particularly simple and suitable for tests of time reversal violation.

1 Introduction

People started to believe in Time Reversal Violation (TRV), since when the first CP violation was discovered, in 1964. This is a consequence of the CPT symmetry, derived under very mild assumptions and confirmed by stringent tests. However till now only one experiment has given evidence for direct TRV. We refer to the CPLEAR experiment[1], which was realized in 1998 and consisted of establishing that the $K^0 \to \overline{K}^0$ transition rate is different than the $\overline{K}^0 \to K^0$ one. Generally it is quite difficult to realize the inverse process to a weak decay and, above all, to compare the two decay rates with enough precision for discovering possible differences.

An alternative way of coping with the problem is to look for theoretical consequences of Time Reversal Invariance (TRI) and to investigate whether these are contradicted by experiment. In particular, let us consider the two non-leptonic decays $K \to \pi\pi$ and $\Lambda \to \pi p$. Such a kind of decays involves strong Final State Interactions (FSI) between the decay products. In the cases considered, given the low mass of the parent resonances, FSI amount essentially to elastic scattering. Then, thanks to the Fermi-Watson theorem, TRI implies the following condition:

$$A = e^{2i\delta} A^*, \tag{1}$$

where A is the weak decay amplitude and δ is the strong phase-shift of elastic scattering. In other words, if one has

$$A = ae^{i(\delta+\phi_w)}, \tag{2}$$

with a real and $\phi_w \neq 0, \pi$, we conclude that Time Reversal (TR) is violated. ϕ_w is generally known as "weak" phase; for example, it could be related to the CP-violating phase which appears in the Cabibbo-Kobayashi-Maskawa (CKM) parametrization of the Yukawa coupling with three-quark generations.

However also this strategy appears impervious, since we are faced with the difficulty of determining phases with sufficient precision. The main aim of our talk is to pose the question as to whether we may be luckier with decays of higher mass resonances, produced copiously at facilities like the LHC. In particular, we shall modify condition (1) for such decays and we shall suggest how to exploit data in order to test TRV.

In section 2 we define the observables which can be extracted in two-body decays with spinning decay products. In section 3 we illustrate how to modify the condition for TRV, when inelastic channels are open. Section 4 is devoted to suggesting tests for TRV. Lastly, a short conclusion is drawn in section 5.

2 Observables in two-body decays

We consider two-body decays involving beauty, and therefore detectable at LHCb, like $\Lambda_b \to \Lambda J/\psi$ and $B \to VV$, V being a vector meson. We examine the observables that can be extracted from such decays. These can be defined, for example, in the so-called helicity frame. To this end we consider a canonical frame, such that the xy-plane coincides with the production plane of the parent resonance. If $\mathbf{p}$ is the momentum of this resonance in the canonical frame, the helicity frame is defined, in the rest frame of one of the two decay products, by three mutually orthogonal unit vectors:

$$\mathbf{e}_L = \frac{\mathbf{p}}{|\mathbf{p}|}, \qquad \mathbf{e}_T = \frac{\mathbf{u} \times \mathbf{e}_L}{|\mathbf{u} \times \mathbf{e}_L|}, \qquad \mathbf{e}_N = \mathbf{e}_T \times \mathbf{e}_L, \tag{3}$$

where $\mathbf{u}$ is the unit vector in the direction of the z-azis of the canonical frame. The observables - angular distribution, polarization components and polarization correlations - can be suitably defined starting from the density matrix of the decay products. We have

$$I(\theta,\phi) = tr\rho, \tag{4}$$

$$I(\theta,\phi)P_i(\theta,\phi) = tr(\rho s_i), \tag{5}$$

$$I(\theta,\phi)P_{ij}(\theta,\phi) = tr(\rho s_i s_j). \tag{6}$$

where ρ is the density matrix of the two decay products, s_i the components of the spin operator, θ and ϕ are respectively the polar and azimuthal angle of the momentum of one of the decay products in the helicity frame and i, j run over the indices L, T, N of the unit vectors. It is worth noting that $P_{ij} \neq P_i P_j$ because of quantum effects.

As an example, we give the expressions of some of the above observables, relative to the decay $\Lambda_b \to \Lambda J/\psi$[2], as functions of the rotationally invariant decay amplitudes, A_{λ_1,λ_2}, λ_1 and λ_2 being the helicities of the two decay products. We have

$$I(\theta,\phi) = \frac{1}{4\pi}(|A_{1/2,0}|^2 + |A_{-1/2,0}|^2 + |A_{1/2,1}|^2 + |A_{-1/2,-1}|^2), \tag{7}$$

$$I(\theta,\phi)P_L^\Lambda(\theta,\phi) = \frac{1}{4\pi}(|A_{1/2,0}|^2 - |A_{-1/2,0}|^2 + |A_{1/2,1}|^2 - |A_{-1/2,-1}|^2), \tag{8}$$

$$I(\theta,\phi)P_{TN}(\theta,\phi) = \frac{1}{4\pi\sqrt{2}} Im(A_{-1/2,-1}A^*_{1/2,0} - A_{-1/2,-1}A^*_{1/2,0}). \tag{9}$$

Really the above formulae hold in the case of unpolarized Λ_b, which is generally not true, since spin-orbit coupling causes a transverse polarization in the production reaction. However, also with this simplifying assumption, it can be shown[2] that one obtains an over-determined linear system, whose unknowns are the products $A_{\lambda_1,\lambda_2}A^*_{\lambda'_1,\lambda'_2}$. Solving this system allows to determine all moduli of the amplitudes and all phases up to one, which we set conventionally to zero. It can be shown that this is a rather general result, valid also, *e. g.*, for decays of the type $0 \to 1 \quad 1$.

3 A condition for TRV with inelastic channels open

Now we generalize eq. (1) to the case when more decay modes are present and therefore inelastic processes are admitted in the scattering between the decay products. In this case TRI implies[3]

$$A_m = S_{mn}A_n^*, \tag{10}$$

where S is the scattering matrix and A_m is the decay amplitude to a given eigenstate of the total angular momentum, denoted by m. The solution to this equation can be obtained by diagonalizing the S-matrix. To this end we recall a result by Suzuki[3], that is,

$$S_{mn} = \sum_k O_{mk} e^{2i\delta_k} O^T_{kn}, \tag{11}$$

where O is an orthogonal matrix and the δ_k are strong phase-shifts. Eqs. (10) and (11) yield

$$A_m = \sum_n O_{mn} a_n e^{i\delta_n}, \tag{12}$$

where a_n are real amplitudes representing the effects of weak interactions on the decay process. Obviously complex values of one or more such amplitudes - that is, "weak" phases - would imply TRV, analogously to the case when only elastic scattering is allowed between the decay products.

We remark that, in a local field theory - like the SM -, even one complex a_n implies CP violation, owing to CPT symmetry. As told in the introduction, in the SM the phase of such a complex amplitude is related to the CKM parametrization.

4 Suggested tests for TRV

Unfortunately the result found in the previous section is generally not useful for detecting TRV. Indeed, first of all, the elements of the matrix O are not known: at best one can elaborate models, as in ref. 3. Secondly, the amplitudes A_n may be determined, at best, up to a phase per decay mode. In fact, as shown in section 2, a decay to spinning and unstable particles allows to determine, through angular distribution, polarizations and polarization correlations[4, 5, 2], all products of the type $A_{\lambda_1\lambda_2}A^*_{\lambda'_1\lambda'_2}$.

These products, in turn, allow to determine all moduli of the amplitudes and their phases relative to a given amplitude, taken as a reference. This is not sufficient to solve the linear system (12) with respect to the products $a_n e^{i\delta_n}$. Therefore generally we cannot elaborate tests for TRV through the method described.

However relation (12) may be considerably simplified, provided we choose suitable decay modes. Let us consider, for example, the decay[2]

$$\Lambda_b \to \Lambda J/\psi. \tag{13}$$

or the following decays of B and B_s, already studied, both theoretically[4, 5, 6, 7] and experimentally[8, 9, 10]:

$$B^+ \;\to\; (J/\psi K^{*+}); \tag{14}$$
$$B^0 \;\to\; (J/\psi K^{*0}); \tag{15}$$
$$B_s \;\to\; (J/\psi\phi), \quad (J/\psi\overline{K}^{*0}). \tag{16}$$

Such decays do not involve isotopic spin, in order to avoid interference among different isospin amplitudes.

We examine the scattering corresponding to FSI between the decay products in decays of the type just illustrated. Then the momentum of the "initial" hadrons in the center-of-mass system is 1.6 to 1.7 GeV/c, therefore the scattering involves at least 15 partial waves. Since the decays considered imply orbital angular momenta not greater than 2, we are faced essentially with central collisions. These give rise mainly to deep inelastic collisions, whose shadow constitutes the largest part of elastic scattering. Instead, anelastic reactions - that is, with excitation of one or both decay products to higher mass states - are suppressed, as well as spin-flip elastic scattering, since these processes occur only in peripheral collisions, so as to keep coherence with the initial state.

But deep inelastic collisions imply a complete loss of coherence with respect to the initial particles. Indeed, any such process includes

a) an exchange of gluons between the two hadrons;

b) hard collisions among partons;

c) parton fragmentations and/or recombinations to hadrons;

d) gluon exchange among final hadrons.

Therefore the sum (12) consists of quite a lot of terms. Moreover it appears logical to assume for the phases in (12) a very rapid dependence on the index n ($n \neq m$), that is, a sudden variation of the phase-shift from state to state. On the contrary, there is no reason for the terms a_n (weak amplitudes) and O_{mn} (kinematic quantities) to depend so strongly on n. Then, for $n \neq m$, the sum can be approximated by the integral of the product of a smooth function times a rapidly varying phase. Therefore only the term with $n = m$ survives in that sum, *i. e.*,

$$A_m = O_{mm} a_m e^{i\delta_m}. \tag{17}$$

This result was obtained also by Wolfenstein[11] with a different line of reasoning.

Eq. (17) can be tested by comparing the decays considered with the CP-conjugate ones and assuming CPT symmetry. Indeed, under this assumption, $\overline{a}_{\overline{m}}$ differs from a_m just by a phase, while O_{mm} and δ_m are CP-invariant, since they depend only on strong interactions. Here and in the following the barred quantities refer to the CP-conjugate process, obviously with opposite helicities, denoted synthetically by $\overline{m}$. Then we have

$$|A_m| = |\overline{A}_{\overline{m}}|, \tag{18}$$

We stress that this test is not mandatory for the method we are going to suggest for detecting TRV.

In order to state tests for TRV, we define, preliminarily, a particular observable that we can extract from analyses of decays, that is, for a given decay mode,

$$\Phi_m = arg(A_m) - arg(A_{m_0}). \tag{19}$$

Here, as explained before, A_{m_0} is conventionally taken to be real. Then we define the following asymmetries:

$$\mathcal{A}_{CP} = \frac{\Phi_m - \overline{\Phi}_{\overline{m}}}{\Phi_m + \overline{\Phi}_{\overline{m}}}, \qquad \mathcal{A}_C = \frac{\Phi_m - \overline{\Phi}_m}{\Phi_m + \overline{\Phi}_m}, \qquad \mathcal{A}_P = \frac{\Phi_m - \Phi_{\overline{m}}}{\Phi_m + \Phi_{\overline{m}}}. \tag{20}$$

Here, again, the barred quantities refer to the phases of the C-conjugated amplitudes.

5 Conclusion

We have shown a theoretical condition which implies TRV. Moreover we have suggested tests to be applied to non-leptonic two-body decays involving spinning particles. The tests may be realized by means of analyses of angular distributions, polarizations and polarization correlations of the decay products. Given the wealth of $b-\overline{b}$ pairs which will be produced at LHC per year (10^{12}), such tests appear to be not so unrealistic. Furthermore, in principle, comparing the results of the suggested tests with the SM predictions could help detecting NP effects, since the condition found is independent of any specific model. Lastly we observe that the condition we require for obtaining evidence for TRV is opposite to the one demanded for detecting CP violations, which needs interference among amplitudes, and therefore, necessarily, more terms in the sum (12)[12].

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