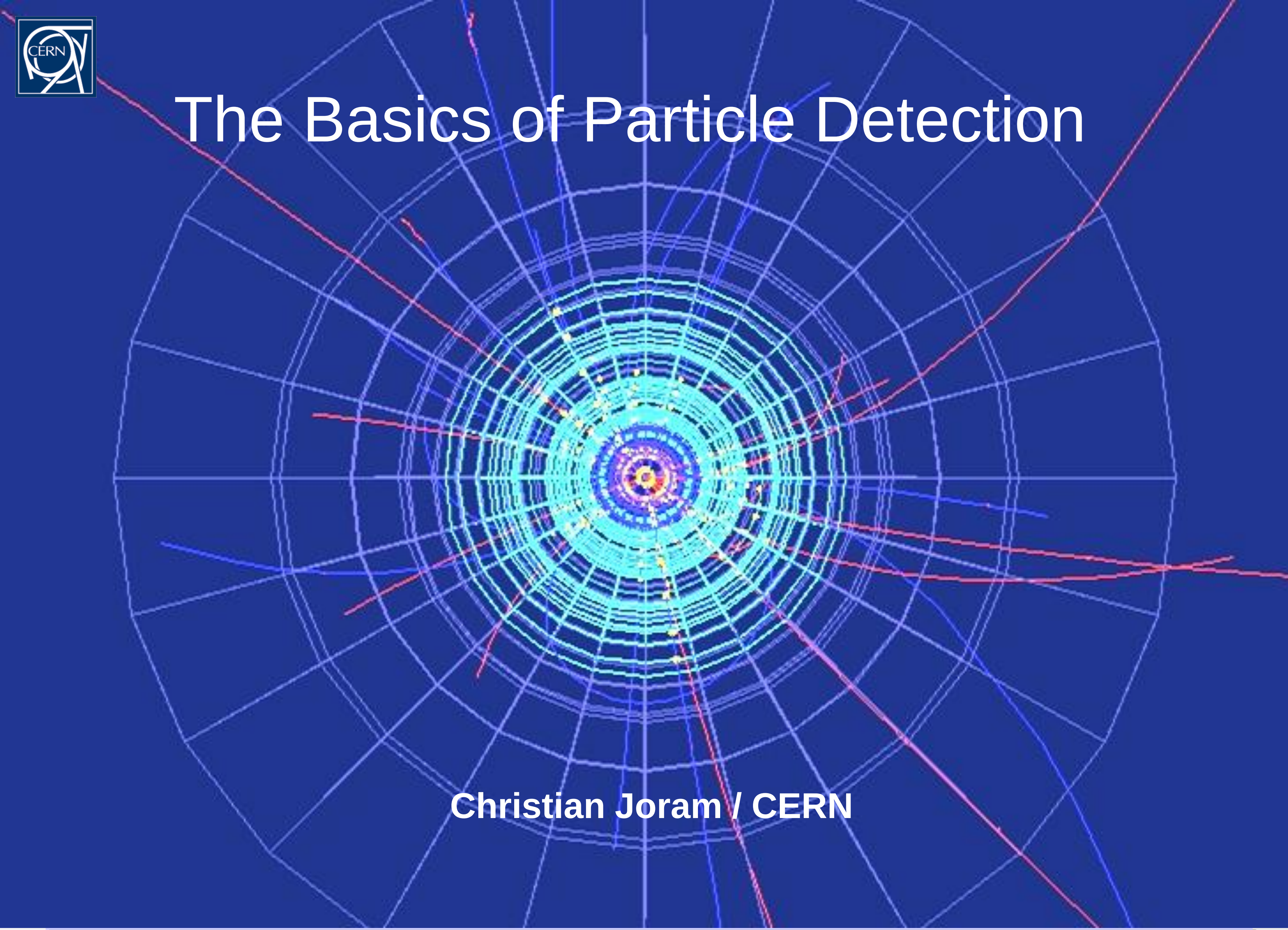
The Basics of Particle Detection



Christian Joram / CERN

Outline

- Lecture 1 – **Interaction of charged particles**
  - Introduction - Some historical examples
  - Units, concept of cross-section, some numbers
  - Particles and interactions
  - Scattering, multiple scattering of charged particles
  - Energy loss by ionization, Bethe-Bloch, Landau
  - Cherenkov and Transition Radiation
- Lecture 2 – **Gaseous and solid state tracking detectors**
- Lecture 3 – **Calorimetry, scintillation and photodetection**

Literature

- **Text books** (a selection)
  - C. Grupen, B. Shwartz, Particle Detectors, 2nd ed., Cambridge University Press, 2008
  - G. Knoll, Radiation Detection and Measurement, 3rd ed. Wiley, 2000
  - W. R. Leo, Techniques for Nuclear and Particle Physics Experiments, Springer, 1994
  - R.S. Gilmore, Single particle detection and measurement, Taylor&Francis, 1992
  - K. Kleinknecht, Detectors for particle radiation , 2nd edition, Cambridge Univ. Press, 1998
  - W. Blum, W. Riegler, L. Rolandi, Particle Detection with Drift Chambers, Springer, 2008
  - R. Wigmans, Calorimetry, Oxford Science Publications, 2000
  - G. Lutz, Semiconductor Radiation Detectors, Springer, 1999

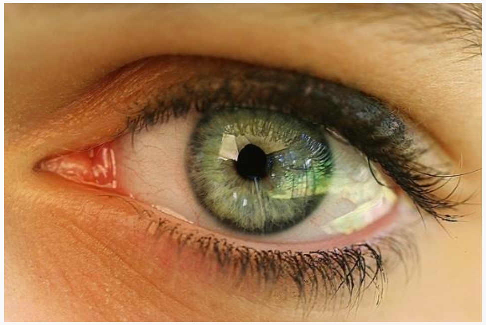
- **Review Articles**
  - Experimental techniques in high energy physics, T. Ferbel (editor), World Scientific, 1991.
  - Instrumentation in High Energy Physics, F. Sauli (editor), World Scientific, 1992.
  - Many excellent articles can be found in Ann. Rev. Nucl. Part. Sci.

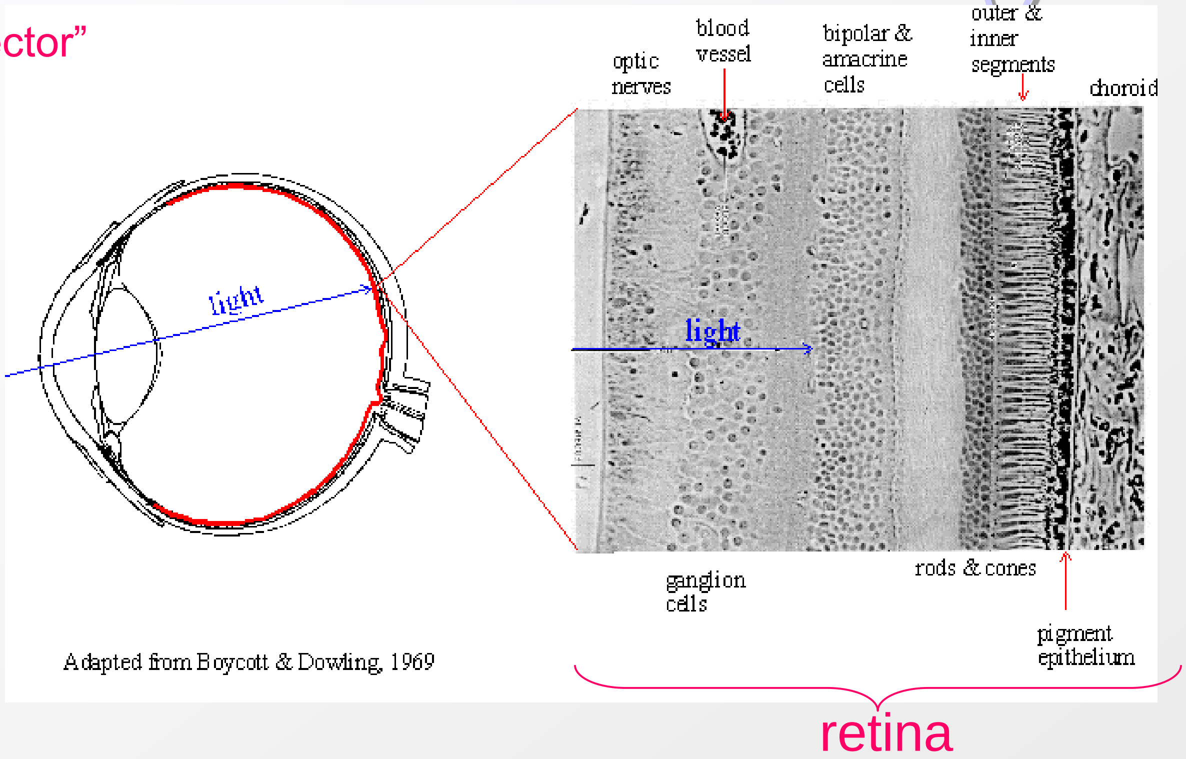
- **Other sources**
  - Particle Data Book, Chin. Phys. C, **40**, 100001 (2016) http://pdg.lbl.gov/pdg.html
  - R. Bock, A. Vasilescu, Particle Data Briefbook
    http://www.cern.ch/Physics/ParticleDetector/BriefBook/ (not updated since 1999)
  - Proceedings of detector conferences (Vienna VCI, TIPP, Elba, IEEE, Como, NDIP)
  - Journals: Nucl. Instr. Meth. A, Journal of Instrumentation

Historical examples

"The oldest particle (photon) detector"

(built many billion times)



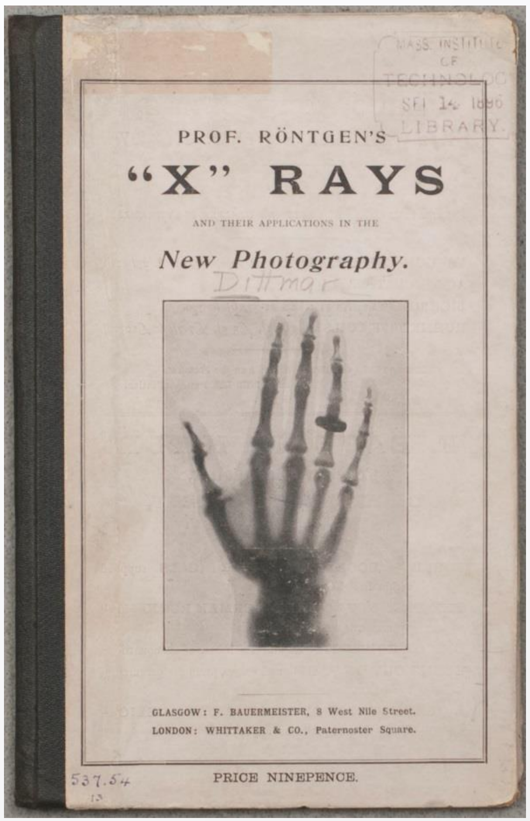


- Good spatial resolution
- Very large dynamic range (1:10^6)

  + automatic threshold adaptation
- Energy (wavelength) discrimination
- Modest sensitivity: 500 to 900 photons must arrive at the eye every second for our brain to register a conscious signal
- Modest speed.

  Data taking rate ~ 10Hz (incl. processing)

Use of photographic paper as detector
→ Detection of photons / x-rays



W. C. Röntgen, 1895: Discovery of the 'X-Strahlen'

Photographic paper/film

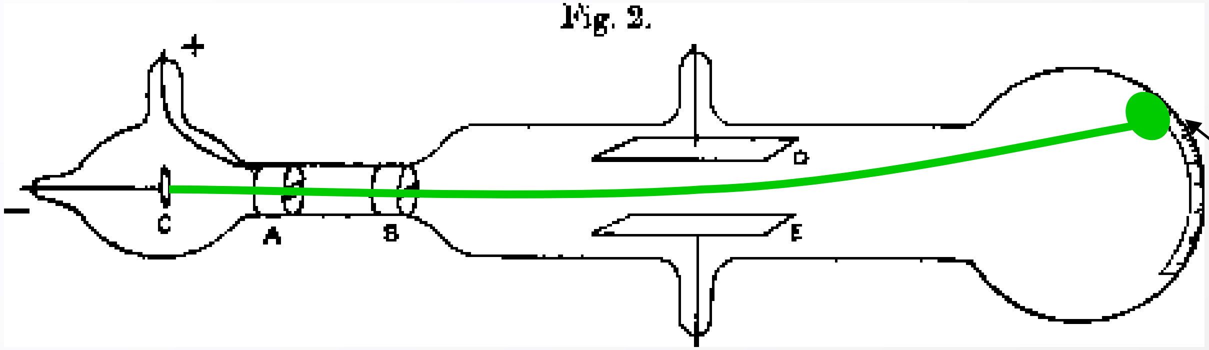
e.g. AgBr / AgCl

AgBr + 'energy'
→ metallic Ag (blackening)

+ Very good spatial resolution
+ Good dynamic range
- No online recording
- No time resolution

cathode ray tube

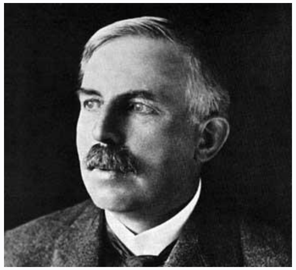
J. Plücker 1858 → J.J. Thomson 1897



From: J.J. Thomson: Cathode Rays.
Philosophical Magazine, **44**, 293 (1897).

"… The rays from the cathode C pass through a slit in the anode A, which is a metal plug fitting tightly into the tube and connected with the earth; after passing through a second slit in another earth-connected metal plug B, they travel between two parallel aluminium plates about 5 cm long by 2 broad and at a distance of 1.5 cm apart; they then fall on the end of the tube and produce a narrow well-defined phosphorescent patch. A scale pasted on the outside of the tube serves to measure the deflexion of this patch…."

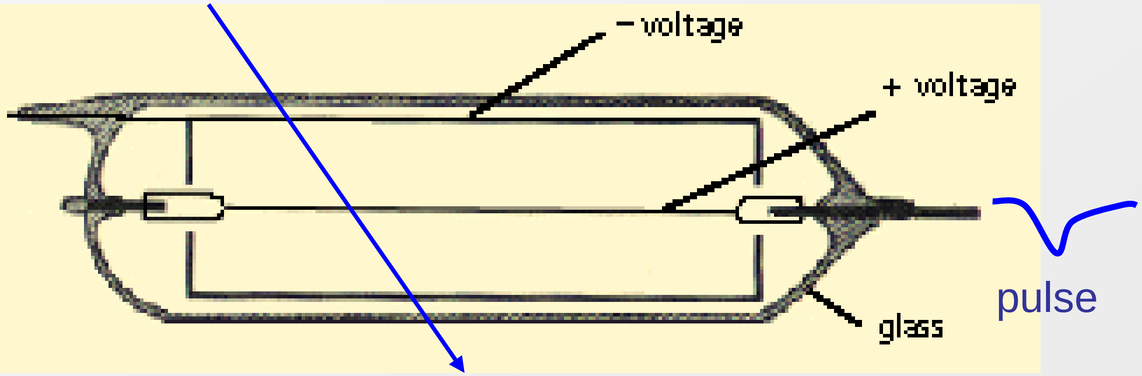
Historical examples



E. Rutherford

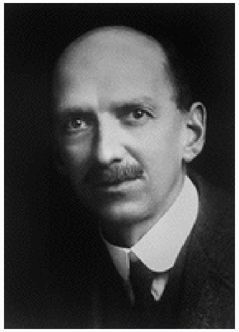
H. Geiger

1909 The Geiger counter, later (1928) further developed and then called Geiger-Müller counter

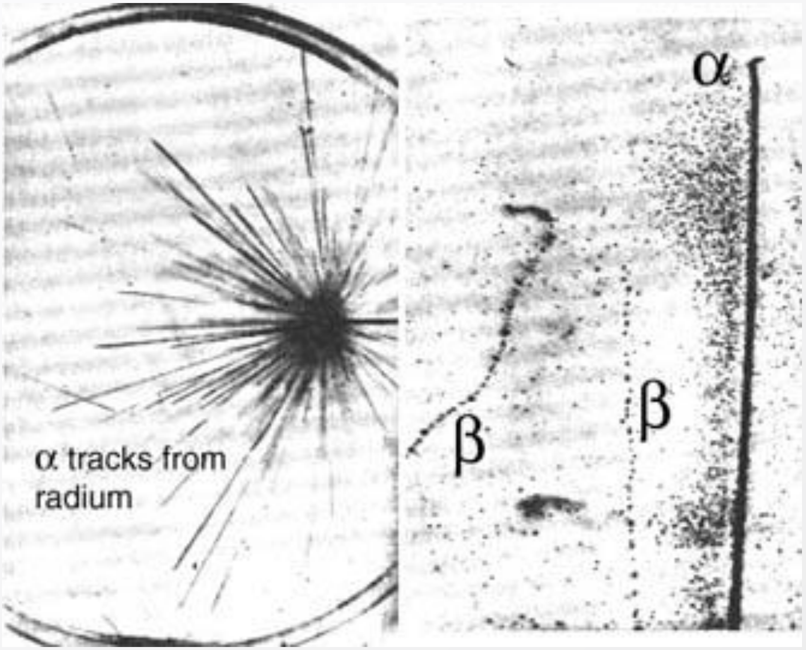


First electrical signal from a particle

1912, Cloud chamber



C. T. R. Wilson



First tracking detector

The general procedure was to allow water to evaporate in an enclosed container to the point of saturation and then lower the pressure, producing a super-saturated volume of air. Then the passage of a charged particle would condense the vapor into tiny droplets, producing a visible trail marking the particle's path.

Detection of particles (and photons) requires an interaction with matter.

The interaction leads to ionisation or excitation of matter.

Most detection interactions are of electromagnetic nature.

In the beginning there were slow "photographic" detectors or fast counting detectors.

New principles, electronics and computers allow us to build fast "photographic" detectors.

Some important definitions and units

$$E^2 = \vec{p}^{\,2}c^2 + m_0^2 c^4$$

- energy E: measure in eV
- momentum p: measure in eV/c
- mass m_o: measure in eV/c^2

$$\beta = \frac{v}{c} \qquad (0 \le \beta < 1) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad (1 \le \gamma < \infty)$$

$$E = m_0 \gamma c^2 \qquad p = m_0 \gamma \beta c \qquad \beta = \frac{pc}{E}$$

1 eV is a tiny portion of energy. 1 eV = $1.6 \cdot 10^{-19}$ J

m_{bee} = 1g = $5.8 \cdot 10^{32}$ eV/c^2

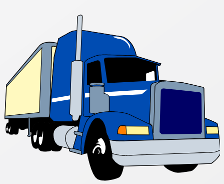
v_{bee} = 1m/s $\rightarrow E_{bee} = 10^{-3}$ J = $6.25 \cdot 10^{15}$ eV

$E_{LHC} = 14 \cdot 10^{12}$ eV

To rehabilitate LHC…

Total stored beam energy: $E_{total} = 10^{14}$ protons $\cdot\ 7 \cdot 10^{12}$ eV $\approx 7 \cdot 10^{26}$ eV $\approx 1 \cdot 10^{8}$ J

this corresponds to a



m_{truck} = 100 T

v_{truck} = 120 km/h

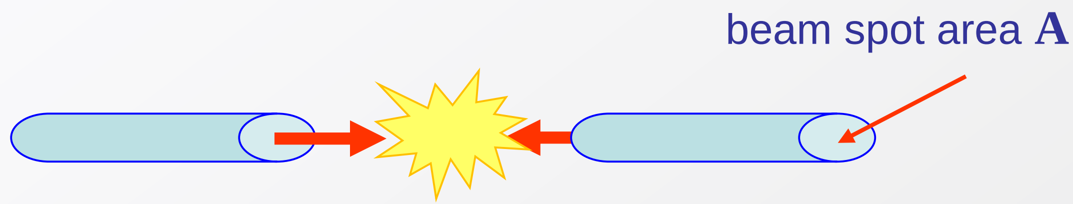
Stored energy in LHC magnets ~ 1 GJ

m_{747} = 400 T

v_{747} = 255 km/h

The concept of cross sections

Cross sections σ or differential cross sections $d\sigma/d\Omega$ are used to express the probability of interactions between elementary particles.



<u>Example:</u> 2 colliding particle beams

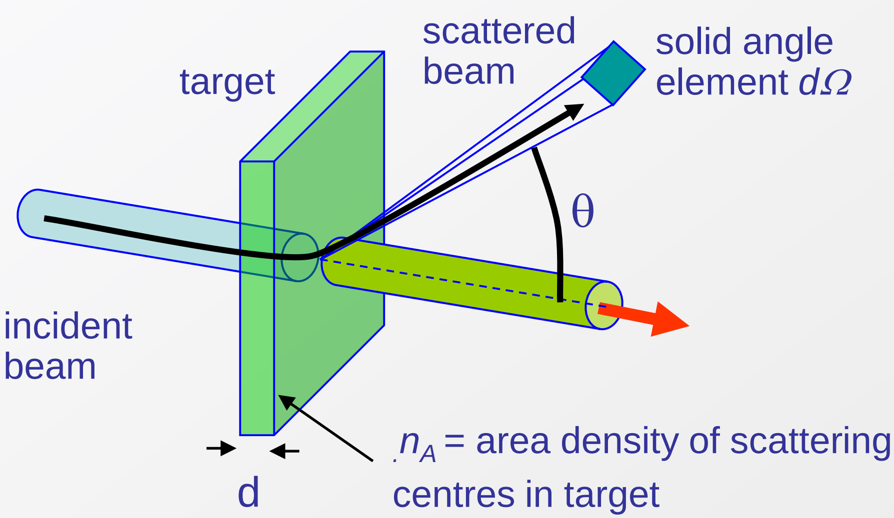
What is the interaction rate $R_{int.}$?

$\Phi_1 = N_1/t$ $\qquad\qquad$ $\Phi_2 = N_2/t$

$$R_{int} \propto \underbrace{N_1 N_2 / (A \cdot t)}_{\text{Luminosity } L \ [\text{cm}^{-2}\ \text{s}^{-1}]} = \sigma \cdot L$$

> σ has dimension area !
> Practical unit:
> 1 barn (b) = 10^{-24} cm^2 = 10^{-28} m^2

<u>Example:</u> Scattering from target

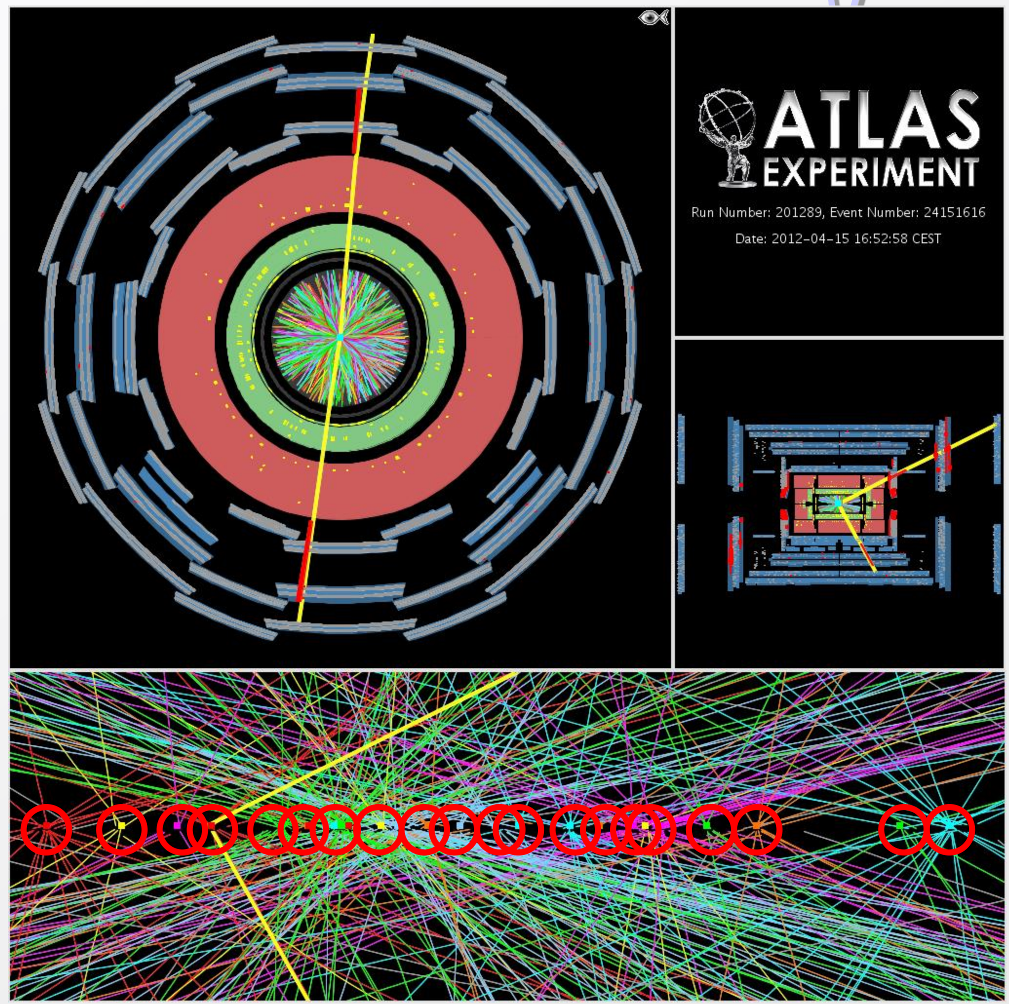


$$N_{scat}(\theta) \propto N_{inc} \cdot n_A \cdot d\Omega$$
$$= d\sigma/d\Omega(\theta) \cdot N_{inc} \cdot n_A \cdot d\Omega$$

$$\left[n_A = \frac{N_A}{A} d \cdot \rho \right]$$
N_A = Avogadro's number
A = mass number

A real event in ATLAS

pp $\rightarrow$ x+Z $\rightarrow$ 2μ + lots of ‘background’



pp collision at $\sqrt{s}$ = 14 TeV,

$\sigma_{total} \approx 100$ mb ~ 10^{-25} cm^2 = 10^{-29} m^2

Reminder: $r_{p,charge}$ ~ 0.85 fm

Geome $\times 10^{-12}$ oss-section:

$A = \pi r^2 = 7 \cdot 10^{-29}$ m^2

We are however interested in processes with $\sigma \approx 10\text{–}100$ fb (Higgs production, new physics).

We need an accelerator with very high luminosity

$L_{LHC} \sim 5 \cdot 10^{33}$ cm^{-2} s^{-1},

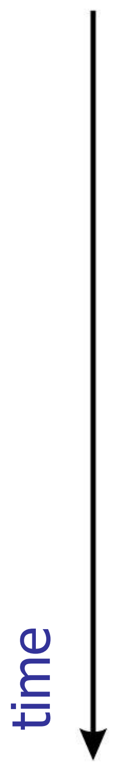
$R = L \cdot \sigma_{total} = 5 \cdot 10^{33} \cdot 10^{-25}$ s^{-1} = $5 \cdot 10^{8}$ s^{-1}

LHC bunch spacing = 50(25) ns

➔ 1 s = $20 \cdot 10^6$ bunch crossings (BC)

➔ In every BC we have on average $5 \cdot 10^8 / 20 \cdot 10^6$ = 25 overlapping pp events !

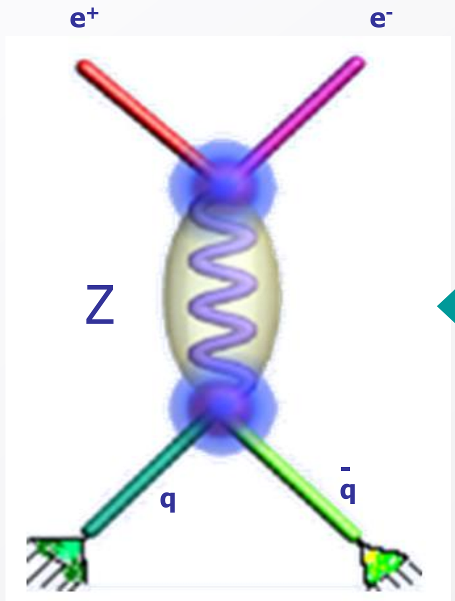
Higgs production (in the macroscopic world)

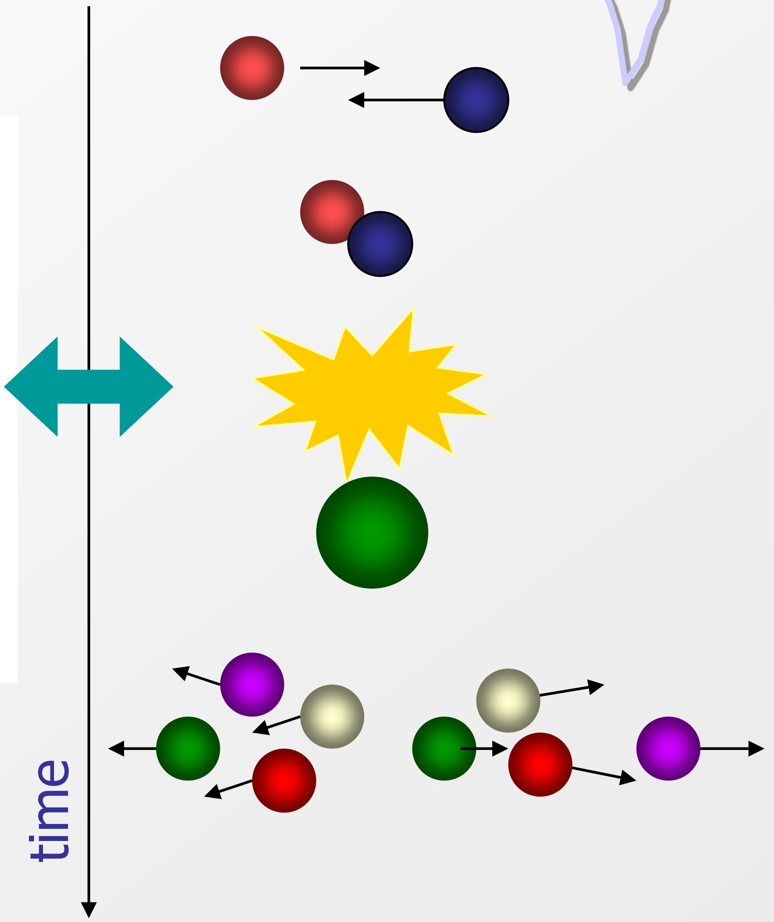


Introduction

Idealistic views of an elementary particle reaction

$$e^{+} + e^{-} \rightarrow Z^{0} \rightarrow q\bar{q}$$
$$(+\mathrm{hadronization})$$





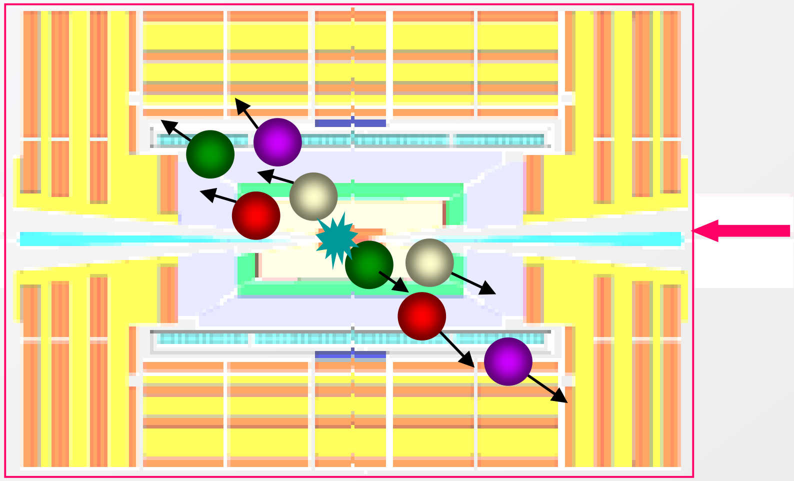
- Usually we can not ‘see’ the reaction itself, but only the end products of the reaction.
- In order to reconstruct the reaction mechanism and the properties of the involved particles, we want the maximum information about the end products !

Introduction

The 'ideal' particle detector should provide…

- coverage of full solid angle (no cracks, fine segmentation)
- detect, track and identify all particles (mass, charge)
- measurement of momentum and/or energy
- fast response, no dead time
- But …. there are practical limitations (technology, space, budget) !

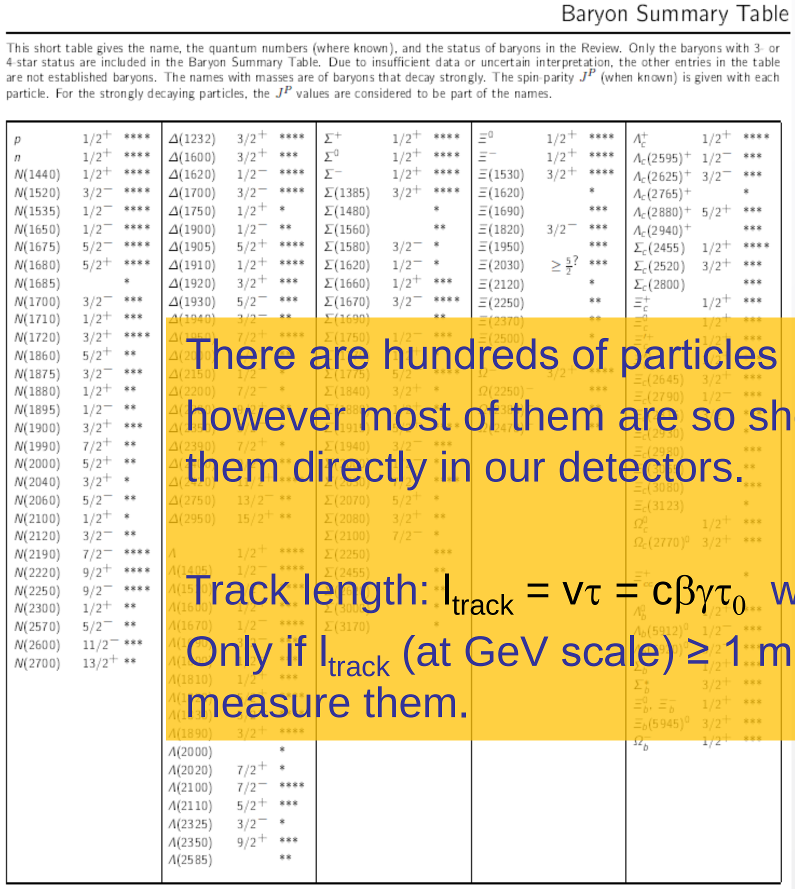
e^+e^-, ep, pp, $p\bar{p}$

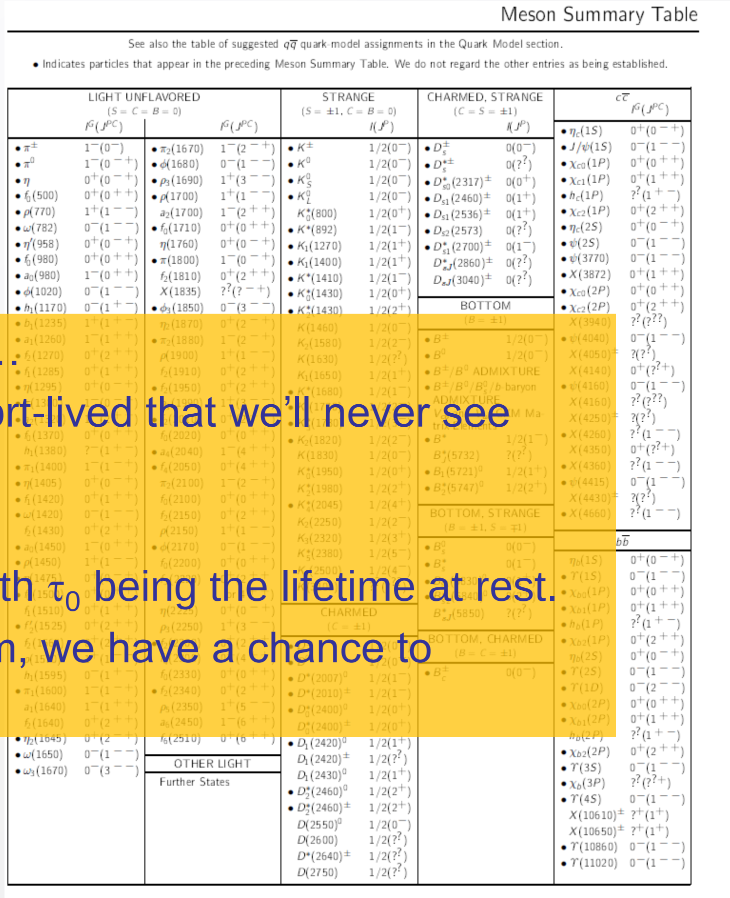


end products
- charged particles
- neutral particles
- photons

- **Particles are detected via their interaction with matter.**
- **Many different physical principles are involved (mainly of electromagnetic nature). Finally we will always observe ionization and/or excitation of matter.**

Introduction





There are hundreds of particles …
however most of them are so short-lived that we'll never see them directly in our detectors.

Track length: $l_{track} = v\tau = c\beta\gamma\tau_0$ with τ_0 being the lifetime at rest.
Only if l_{track} (at GeV scale) ≥ 1 mm, we have a chance to measure them.

Leptons

- e
- μ
- τ
- ν_e
- ν_μ
- ν_τ

Gauge bosons

- γ
- $W^{+/-}$
- Z
- g
- H

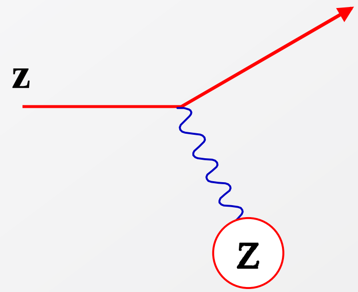
Which are left then? These 8 particles (and their antiparticles).

	γ	p	n	$e^\pm$	$\mu^\pm$	$\pi^\pm$	$K^\pm$	K_0 (K_S/K_L)
τ_0	∞	∞	∞	∞	2.2 μs	26 ns	12 ns	89 ps / 51 ns
l_{track} (p=1GeV)	∞	∞	∞	∞	6.1 km	5.5 m	6.4 m	5 cm / 27.5 m

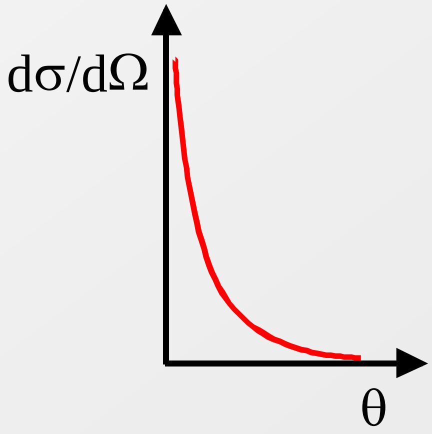
Interaction of charged particles

(Elastic) Scattering

An incoming particle with charge z interacts elastically with a target of nuclear charge Z.
The cross-section for this e.m. process is



$$\frac{d\sigma}{d\Omega}(\theta) = 4zZr_e^2\left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{\sin^4\theta/2}$$ Rutherford formula



- Approximation
  - Non-relativistic
  - No spins

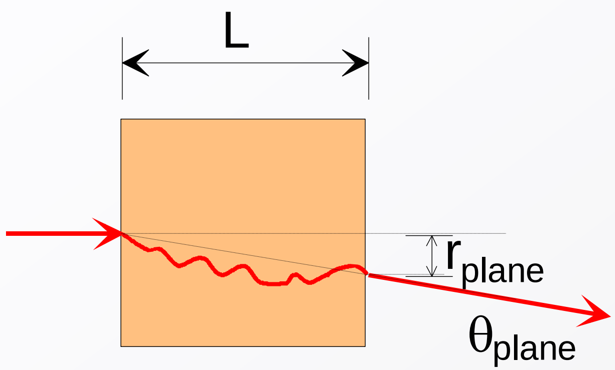
- Average scattering angle $\langle\theta\rangle = 0$
- Cross-section for $\theta \to 0$ infinite !

- Scattering does not lead to significant energy loss (nuclei are heavy!)

Interaction of charged particles

In a sufficiently thick material layer a particle will undergo …

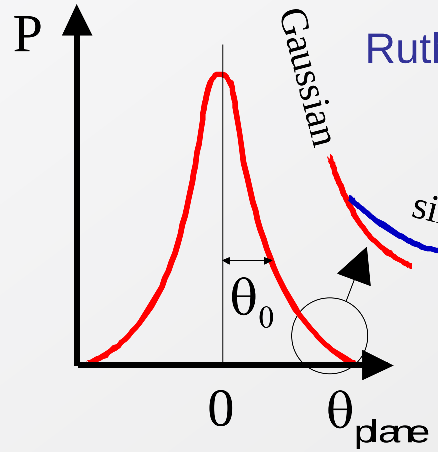
- **Multiple Scattering**



The final displacement and direction are the result of many independent random scatterings

→ Central limit theorem

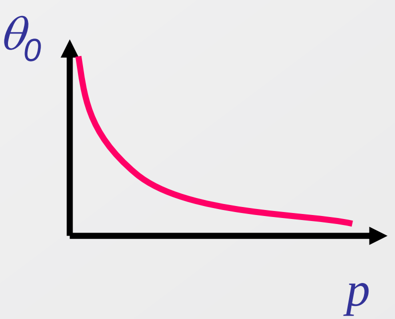
→ Gaussian distribution

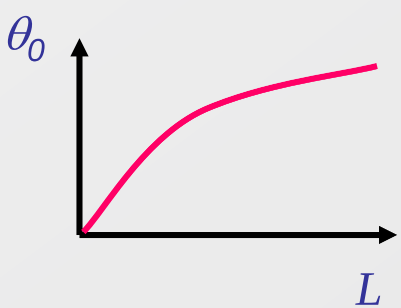


$$\theta_0 = \theta_{plane}^{RMS} = \sqrt{\langle \theta_{plane}{}^2 \rangle} = \frac{1}{\sqrt{2}} \theta_{space}^{RMS}$$

Approximation $\theta_0 \propto \frac{1}{p} \sqrt{\frac{L}{X_0}}$

X_0 is radiation length of the medium (discuss later)

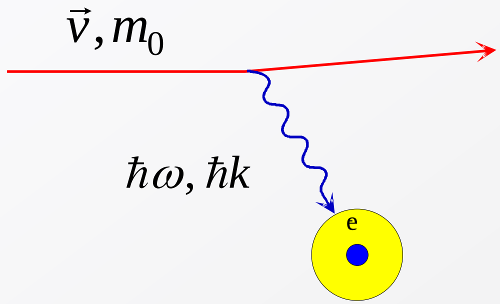




Detection of charged particles

Particles can only be detected if they deposit energy in matter.
How do they lose energy in matter ?

Discrete collisions with the atomic electrons of the absorber material.



$$\left\langle \frac{dE}{dx} \right\rangle = -\int_0^\infty NE \frac{d\sigma}{dE} \hbar \, d\omega$$

N : electron density

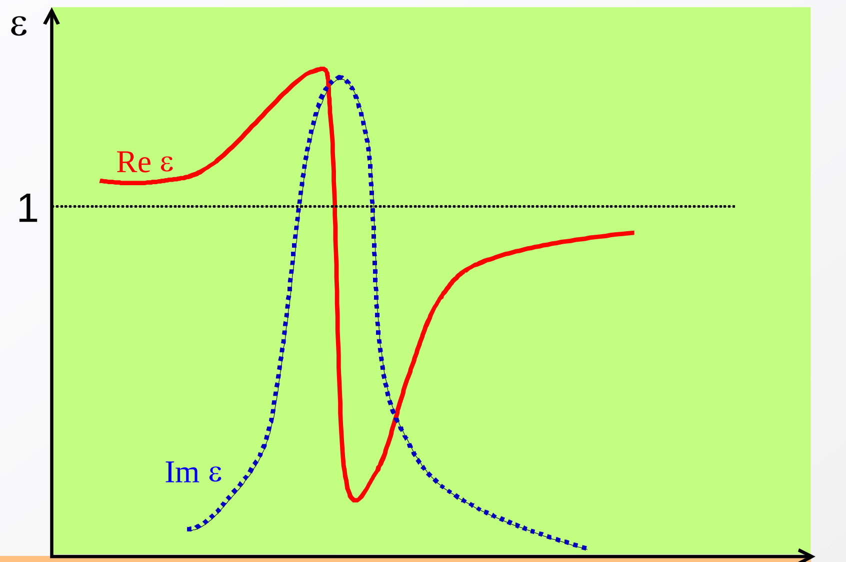
$$\left(\omega = 2\pi f = 2\pi \cdot \frac{c}{\lambda} \right)$$

W.W.M. Allison, J.H. Cobb,
Ann. Rev. Nucl. Part. Sci. 1980. 30: 253-98

Collisions with nuclei not important for energy loss ($m_N >> m_e$)

If $\hbar\omega, \hbar k$ are in the right range ⇨ ionization.

Interaction of charged particles



regime:	optical	absorptive	X-ray
effect:	Cherenkov radiation	ionisation	transition radiation

Optical behaviour of medium is characterized by the complex dielectric constant ε

$\mathrm{Re}\sqrt{\varepsilon} = n$ Refractive index

$\mathrm{Im}\,\varepsilon = k$ Absorption parameter

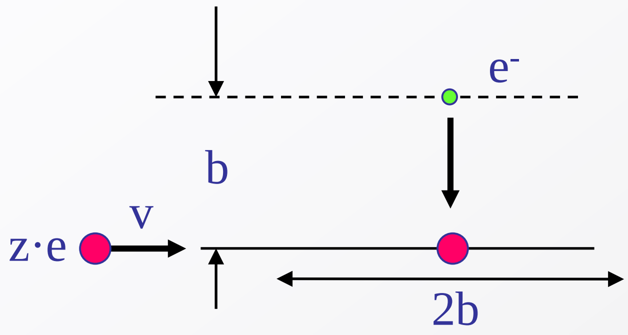
Instead of ionizing an atom or exciting the matter, under certain conditions the photon can also escape from the medium.
⇨ Emission of **Cherenkov** and **Transition** radiation. (See later). This emission of real photons contributes also to the energy loss.

Interaction of charged particles

- **Average differential energy loss** $\left\langle \frac{dE}{dx} \right\rangle$

… making Bethe-Bloch plausible.

Energy loss at a single encounter with an electron



$$F_c = \frac{ze^2}{b^2} \qquad \Delta t = \frac{2b}{v} \qquad \Delta p_e = F_c \Delta t$$

$$\Delta E_e = \frac{(\Delta p_e)^2}{2m_e} = \frac{2z^2e^4}{b^2v^2m_e} = \frac{2r_e^2 m_e c^2 z^2}{b^2} \cdot \frac{1}{\beta^2}$$

Introduced classical electron radius $\quad r_e = \frac{e^2}{m_e c^2}$

How many encounters are there ?

Should be proportional to electron density in medium $\quad N_e \propto \frac{Z}{A} N_A \cdot \rho$

The real Bethe-Bloch formula.

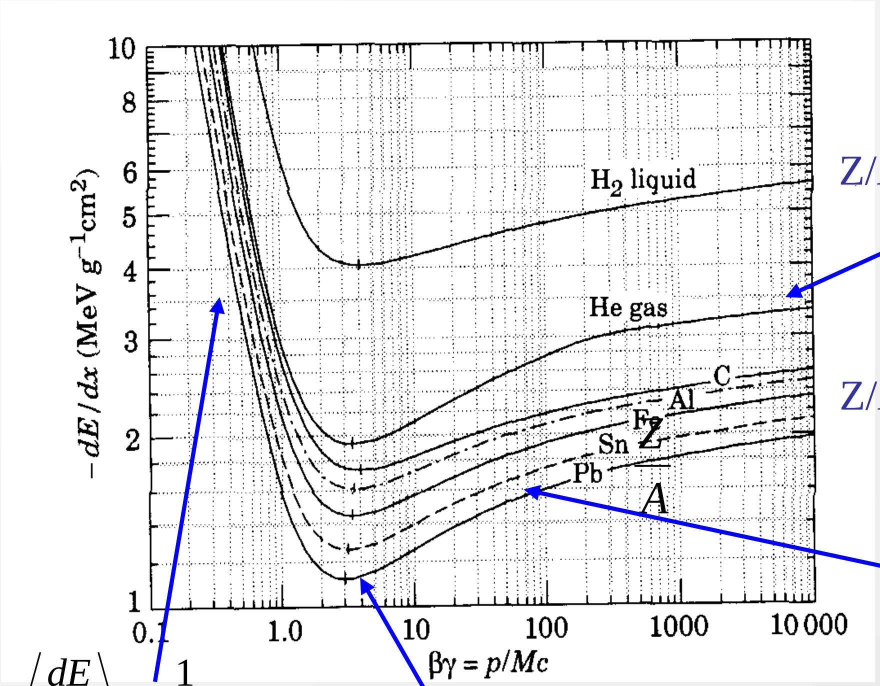
$$\left\langle \frac{dE}{dx} \right\rangle = -4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \gamma^2 \beta^2}{I^2} T^{\max} - \beta^2 - \frac{\delta}{2} \right]$$

Interaction of charged particles

Energy loss by Ionisation only → Bethe - Bloch formula

$$\left\langle \frac{dE}{dx} \right\rangle = -4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \gamma^2 \beta^2}{I^2} T^{max} - \beta^2 - \frac{\delta}{2} \right]$$

- *dE/dx* in [MeV g^{-1} cm^2]
- Strictly valid for "heavy" particles ($m \geq m_\mu$) only. For electrons, use Berger Seltzer formula
- *dE/dx* depends only on β, independent of m !
- First approximation: medium simply characterized by *Z/A* ~ electron density



Z/A = 1

"Fermi plateau"

Z/A~0.5

$\frac{Z}{A}$

$$\left\langle \frac{dE}{dx} \right\rangle \propto \ln \beta^2\gamma^2$$

"relativistic rise"

$$\left\langle \frac{dE}{dx} \right\rangle \propto \frac{1}{\beta^2}$$

"kinematical term"

$\beta\gamma \approx$ 3-4
minimum ionizing particles, MIPs

Useful link: http://www.nist.gov/pml/data/star/

Interaction of charged particles

Bethe - Bloch formula cont'd

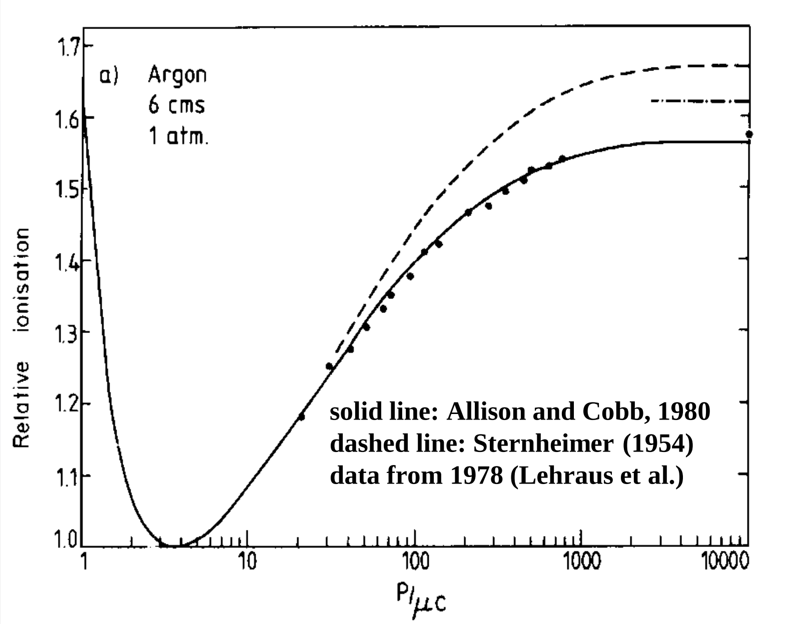
$$\left\langle \frac{dE}{dx} \right\rangle = -4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \gamma^2 \beta^2}{I^2} T^{max} - \beta^2 - \frac{\delta}{2} \right]$$

- relativistic rise - $\ln \gamma^2$ term - attributed to relativistic expansion of transverse E-field → contributions from more distant collisions.
- relativistic rise cancelled at high γ by "density effect", polarization of medium screens more distant atoms. Parameterized by δ (material dependent) → Fermi plateau
- Formula takes into account energy transfers

$$I \leq dE \leq T^{max} \qquad I \approx I_0 Z \;\; \text{with} \;\; I_0 = 10\,\text{eV}$$

I: mean ionization potential, measured (fitted) for each element.

Measured and calculated dE/dx



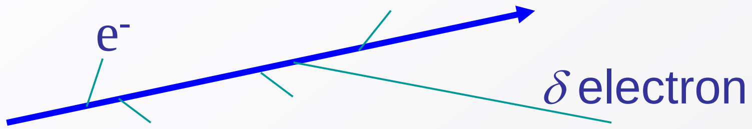
Interaction of charged particles

Real detector (limited granularity) can not measure <*dE/dx*> !

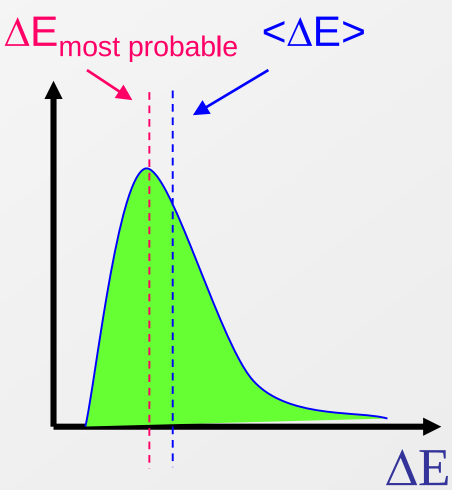
It measures the energy ΔE deposited in a layer of finite thickness δx.

For thin layers or low density materials:

→ Few collisions, some with high energy transfer.



→ Energy loss distributions show large fluctuations towards high losses: ”Landau tails”

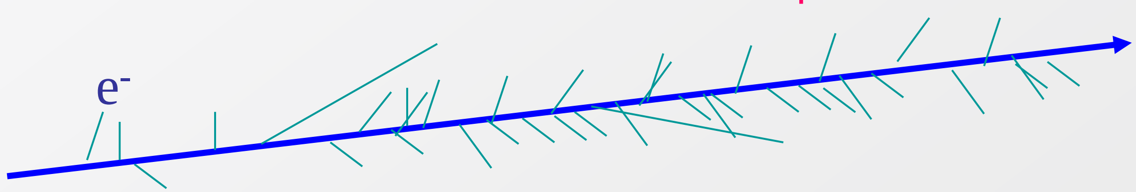


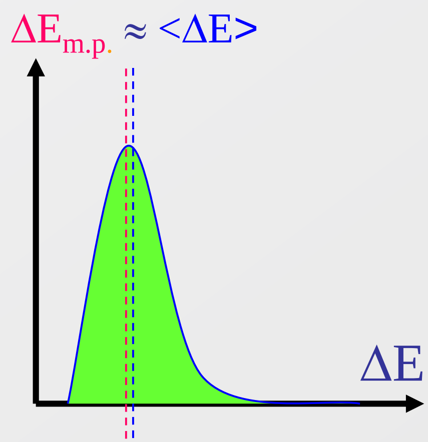
Example: Si sensor: 300 μm thick. $\Delta E_{\text{most probable}}$ ~ 82 keV   <ΔE> ~ 115 keV

For thick layers and high density materials:

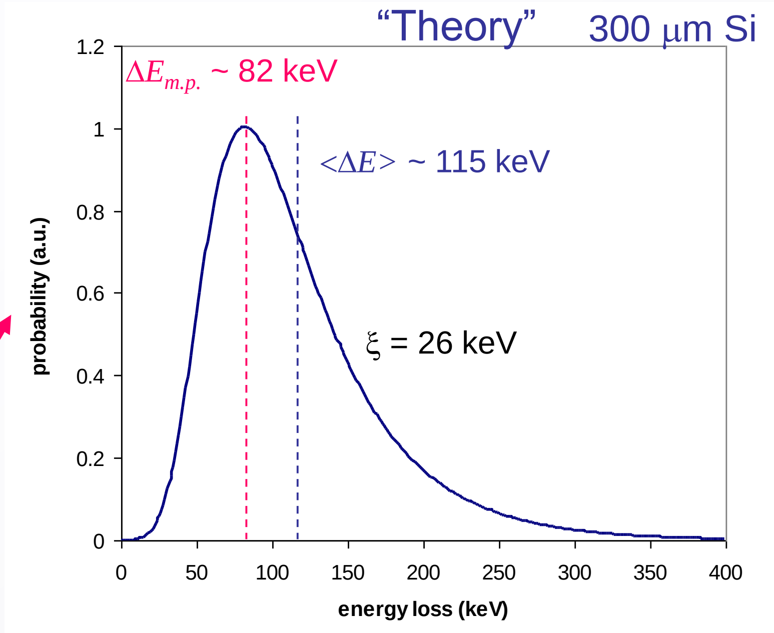
→ Many collisions.

→ Central Limit Theorem → Gaussian shaped distributions.





Interaction of charged particles



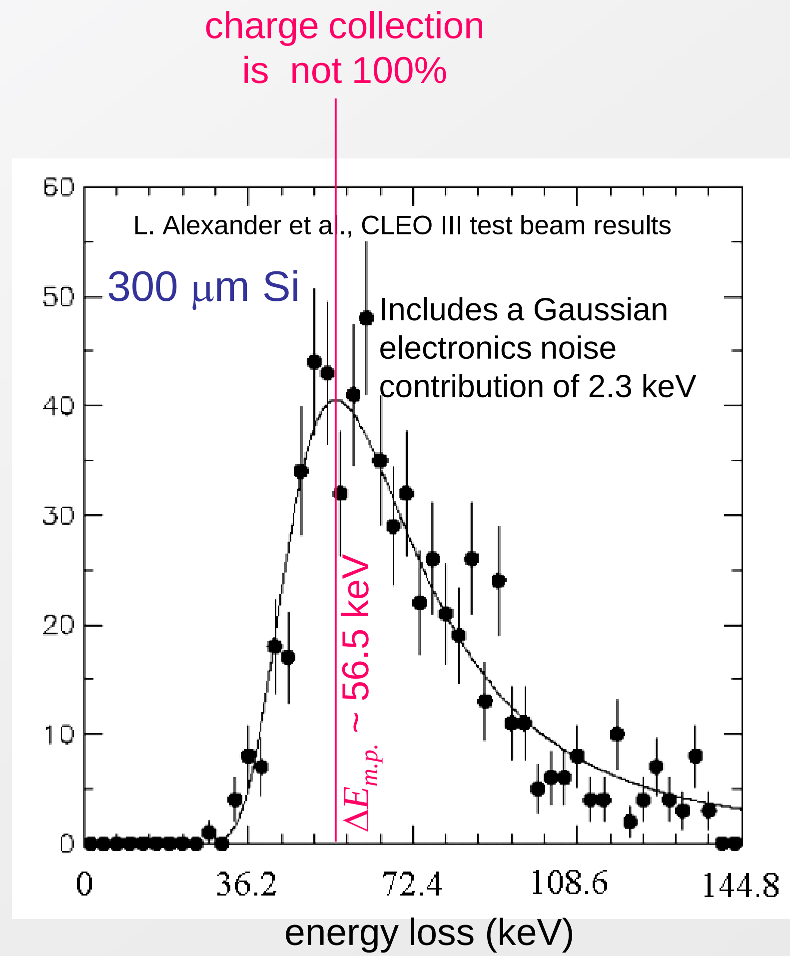
Landau's theory J. Phys (USSR) 8, 201 (1944)

$$f(x,\Delta E) = \frac{1}{\xi}\Omega(\lambda) \qquad \Omega(\lambda) \approx \frac{1}{\sqrt{2\pi}}\exp\left\{-\tfrac{1}{2}\left(\lambda + e^{-\lambda}\right)\right\}$$

$$\lambda = \frac{\Delta E - \Delta E_{m.p.}}{\xi}$$

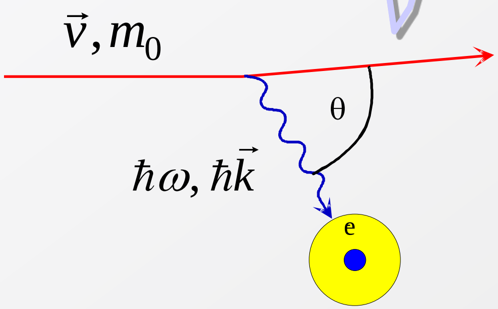
$$\xi = \frac{2\pi N e^4}{m_e v^2}\frac{Z}{A}x$$

x (300 μm Si) = 69 mg/cm^2



… more interactions of charged particles

In addition to ionisation there are other ways of energy loss !



A photon in a medium has to follow the dispersion relation

$$\omega = 2\pi f = 2\pi \frac{c/n}{\lambda} = k\frac{c}{n} \qquad \omega^2 - \frac{k^2c^2}{\varepsilon} = 0 \qquad \varepsilon = n^2$$

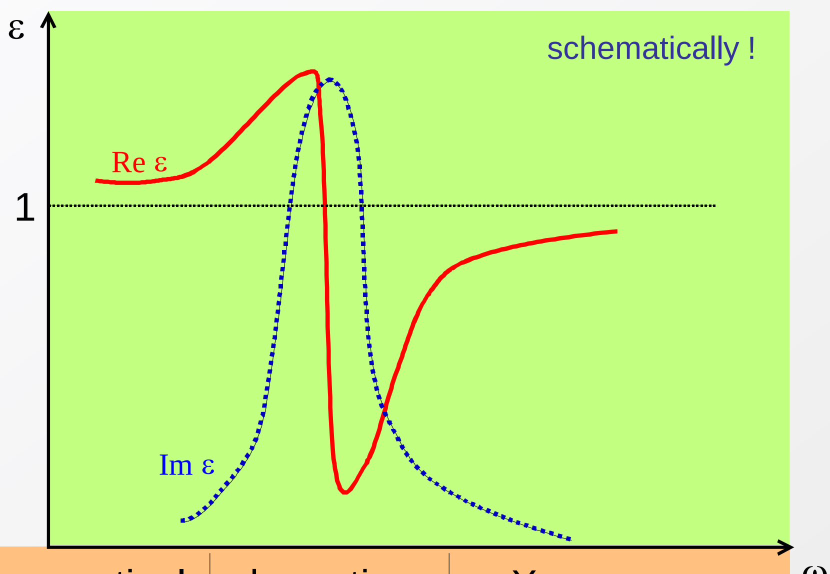
Assuming soft collisions + energy and momentum conservation

$$E = E' + \hbar\omega \qquad \vec{p} = \vec{p}\,' + \hbar\vec{k}$$

$\rightarrow$ emission of real photons:

$$\omega \cong \vec{v}\cdot\vec{k} = v \cdot k \cos\theta$$

$$\rightarrow \quad \cos\theta = \frac{\omega}{vk} = \frac{kc}{n}\cdot\frac{1}{vk} = \frac{1}{n\beta}$$



regime:	optical	absorptive	X-ray
effect:	Cherenkov radiation	ionisation	transition radiation

Emission of photons if

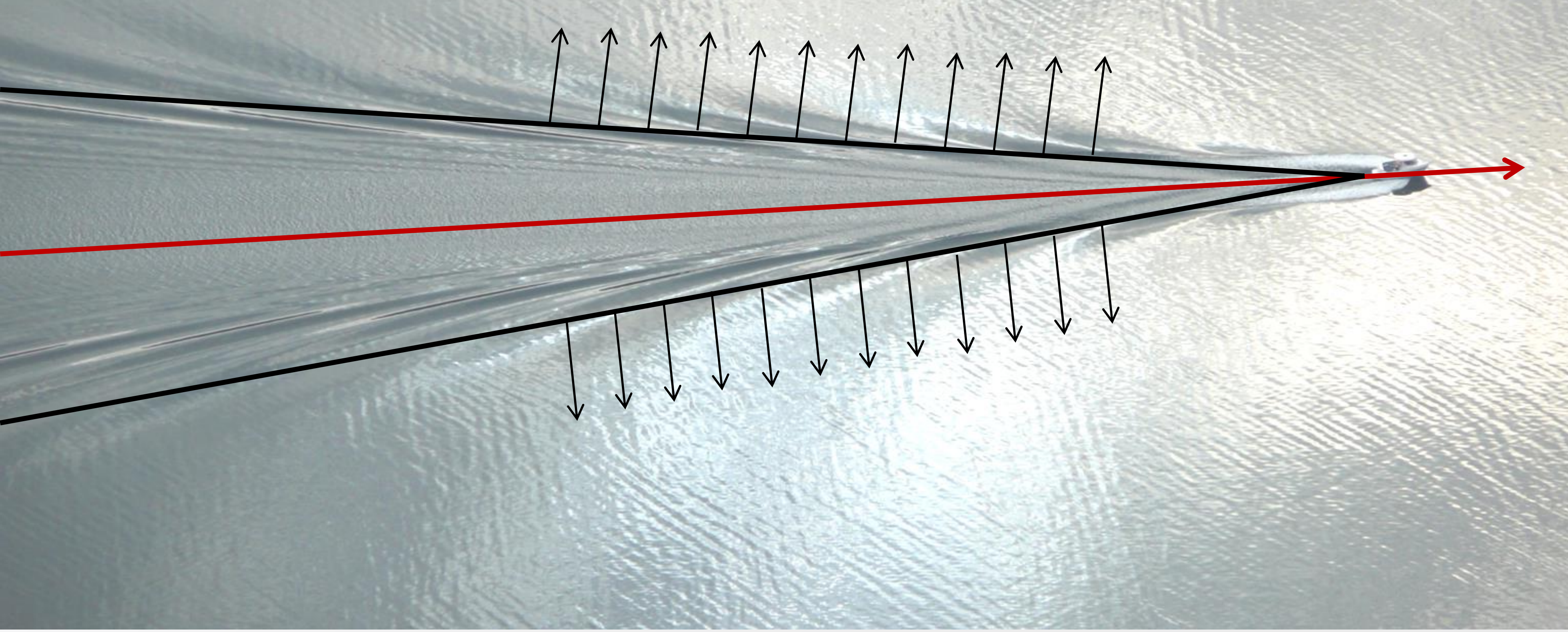
$$\beta = \frac{1}{n\cdot\cos\theta} \qquad \beta \geq 1/n \qquad v \geq c/n$$

A particle emits real photons in a dielectric medium if its speed $v = \beta \cdot c$ is greater than the speed of light in the medium c/n

Cherenkov Radiation

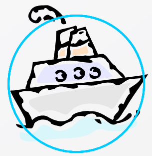
A charged particle, moving though a medium at a speed which is greater than the speed of light in the medium, produces Cherenkov light.

Classical analogue: <u>fast</u> boat on water

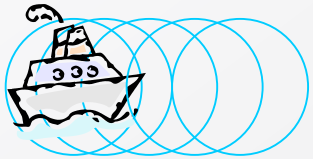


Propagating waves

- A stationary boat bobbing up and down on a lake, producing waves

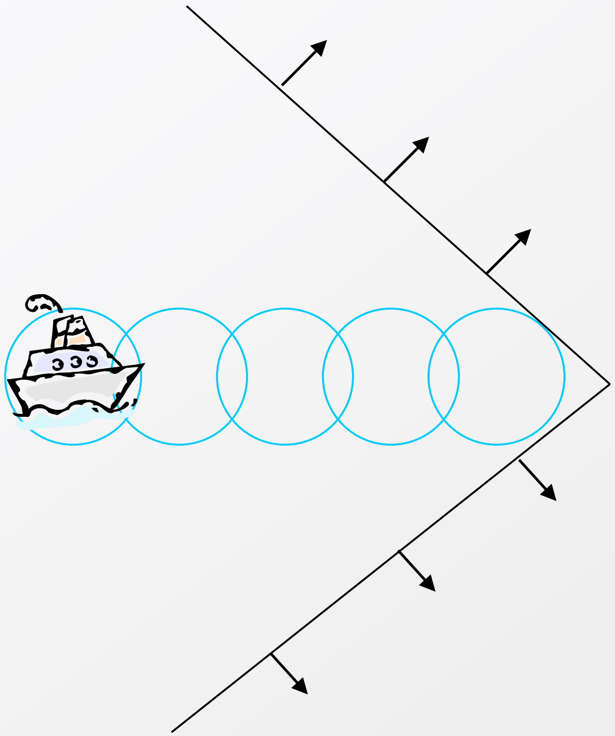


- Now the boat starts to move, but slower than the waves



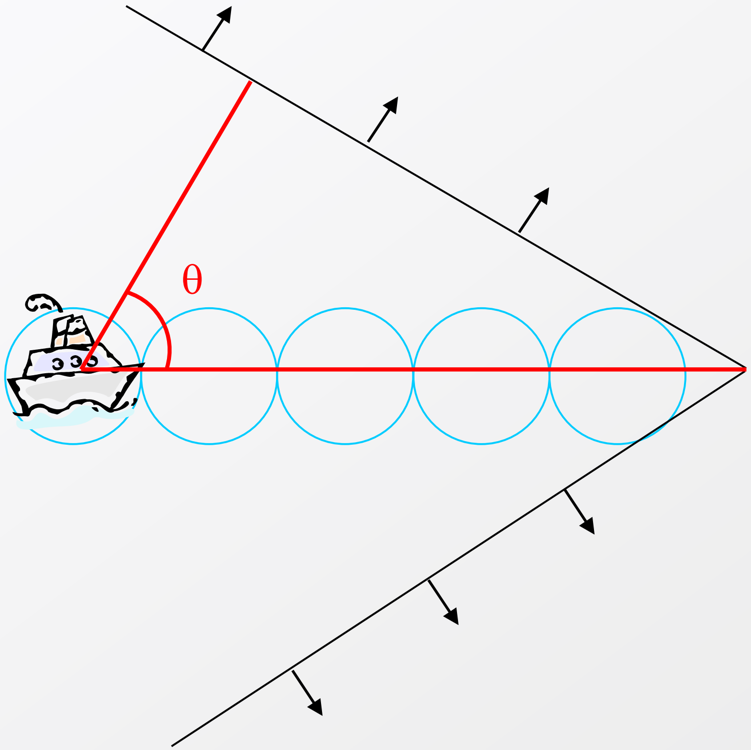
No coherent wavefront is formed

- Next the boat moves faster than the waves



A coherent wavefront is formed

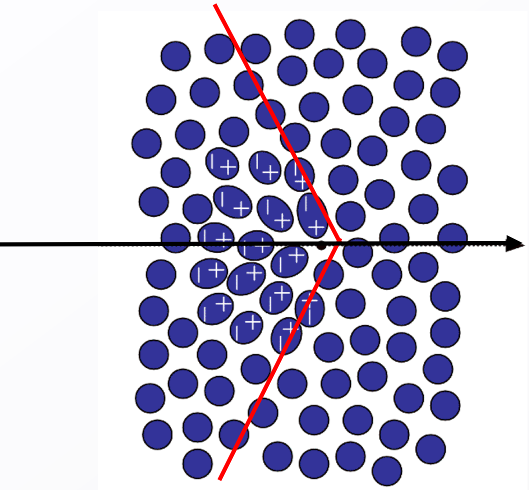
- Finally the boat moves even faster



The angle of the coherent wavefront changes with the speed

$$\cos\theta = v_{\text{wave}} \ / \ v_{\text{boat}}$$

… back to Cherenkov radiation

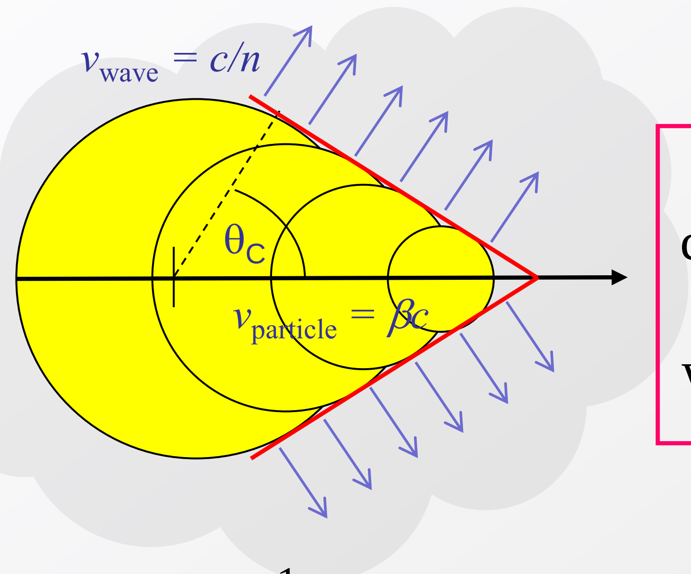


The dielectric medium is polarized by the passing particle.

↘ "the radiator"

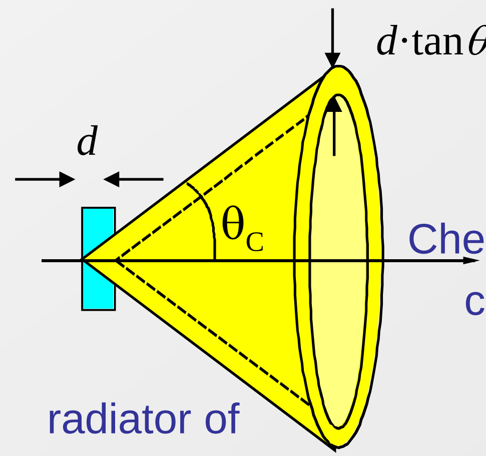
A coherent wave front forms if $v_{\text{particle}} \geq c_{\text{medium}} = \dfrac{c}{n}$

$$\beta_{\text{particle}} \geq \frac{1}{n} \qquad (n = \text{refr. index})$$



$$\cos\theta_C = \frac{v_{\text{wave}}}{v_{\text{particle}}} = \frac{1}{n\beta}$$

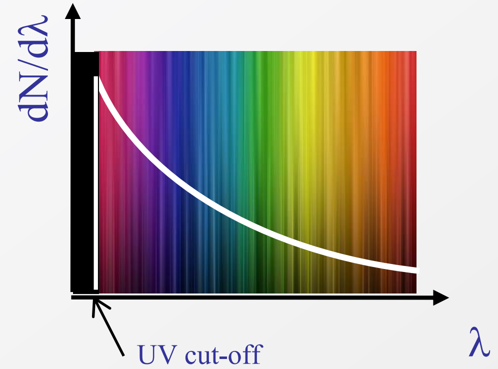
with $n = n(\lambda) \geq 1$



- $\beta_{thr} = \dfrac{1}{n} \;\rightarrow\; \theta_C \approx 0$ Cherenkov threshold
- $\theta_{\max} = \arccos\dfrac{1}{n}$ 'saturated' angle (β=1)

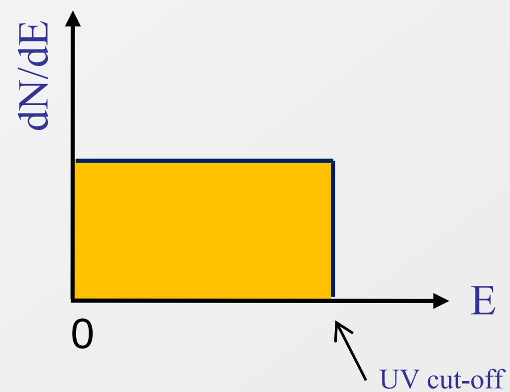
Number of emitted photons per unit length and unit wavelength/energy interval

$$\frac{d^2N}{dx d\lambda} = \frac{2\pi z^2\alpha}{\lambda^2}\left(1-\frac{1}{\beta^2 n^2}\right) = \frac{2\pi z^2\alpha}{\lambda^2}\sin^2\theta_C$$



$$\frac{d^2N}{dx d\lambda} \propto \frac{1}{\lambda^2} \quad \text{with} \quad \lambda = \frac{c}{f} = \frac{hc}{E} \qquad \frac{d^2N}{dx dE} = const.$$

$$\frac{dN}{dx} = 370/\text{cm}\,\sin^2\theta\cdot\Delta E_{\text{detector}}$$



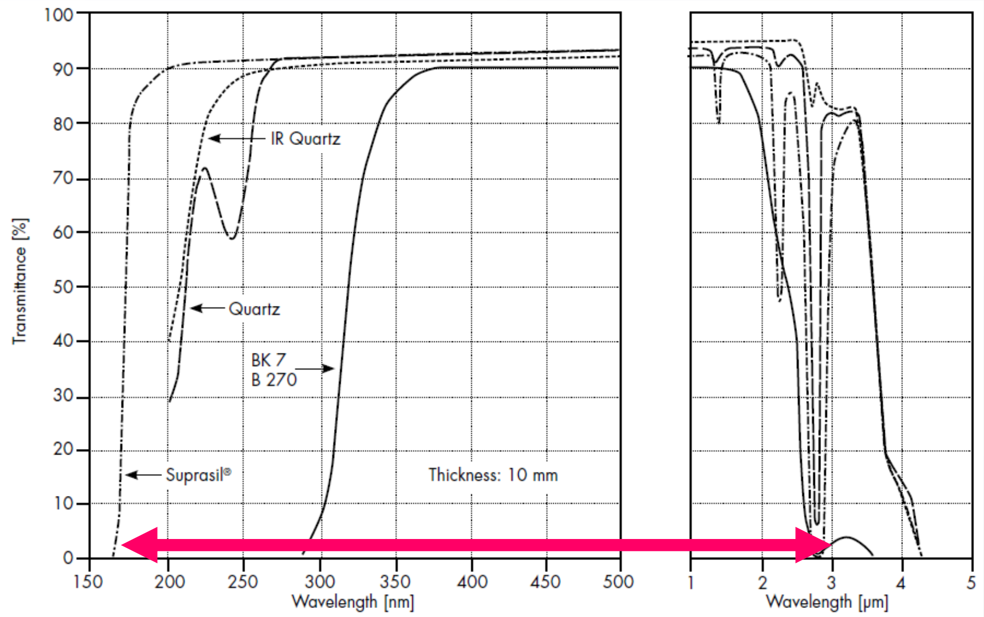
Cherenkov effect is a weak light source. There are only few photons produced.

$$\left.\frac{dE}{dx}\right|^{\text{Cherenkov}} <\approx 1\,\text{keV/cm} \approx 0.001\cdot\left.\frac{dE}{dx}\right|^{\text{Ionization}}$$

Cherenkov radiation

Estimate the energy loss by Cherenkov radiation in quartz

$$\frac{dN_\gamma}{dx} = \frac{\alpha}{\hbar c}\sin^2\theta\cdot\Delta E = \frac{370}{eV\cdot cm}\sin^2\theta\cdot\Delta E$$



$n_{quartz} \approx 1.46$

→ $\sin^2\theta = 1 - \cos^2\theta = 1 - 1/n^2 = 0.53$

$\lambda_{min} = 160$ nm → $E_{max} \approx 1234 / \lambda \approx 7.7$ eV

$\lambda_{max} \approx 3$ µm = 3000 nm → $E_{min} \approx 0$

$\Delta E = 7.7$ eV $\quad \langle E_\gamma \rangle = \Delta E/2 = 3.85$ eV

$$\frac{dN_\gamma}{dx} = 370\cdot 0.53\cdot 7.7 = 1509 \text{ photons/cm}$$

$$\left.\frac{dE}{dx}\right|^{\text{Cherenkov}} = 1509\cdot\langle E_\gamma\rangle = 5.8\text{ keV/cm} \quad\leftrightarrow\quad \left.\frac{dE}{dx}\right|^{\text{Ionization}} \approx \rho_{\text{quartz}}\cdot 2\text{ MeV/g}\cdot\text{cm}^2 \approx 5\text{ MeV/cm}$$

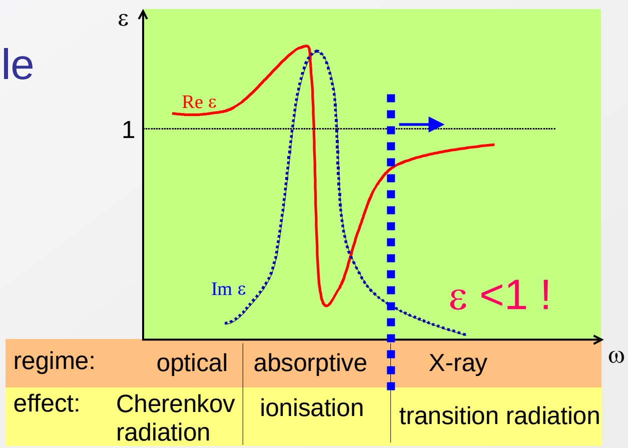
Cherenkov effect is a weak but very useful light source.

Transition Radiation

TR is also called sub-threshold Cherenkov radiation

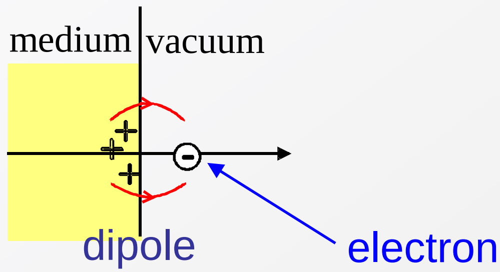
Transition Radiation was predicted by Ginzburg and Franck in 1946

Relativistic theory: G. Garibian, Sov. Phys. JETP63 (1958) 1079



TR is electromagnetic radiation emitted when a charged particle traverses a medium with a discontinuous refractive index, e.g. the boundaries between vacuum and a dielectric layer.

A (too) simple picture



Medium gets polarized. Electron density displaced from its equilibrium
→ Dipole, varying in time → radiation of energy.

Radiated energy per medium/vacuum boundary:

$$W = \frac{1}{3}\alpha\hbar\omega_p\gamma \qquad \omega_p = \sqrt{\frac{N_e e^2}{\varepsilon_0 m_e}} \quad \begin{pmatrix}\text{plasma}\\ \text{frequency}\end{pmatrix} \qquad \hbar\omega_p \approx 20\,\text{eV} \;\; (\text{plastic radiators})$$

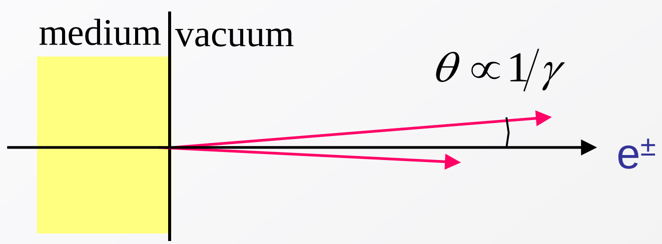
Transition Radiation

Radiated energy per medium/vacuum boundary:

$$W = \frac{1}{3}\alpha\hbar\omega_p\gamma$$

$W \propto \gamma$ → only high energetic $e^{\pm}$ emit TR of detectable intensity. → particle ID

- Lorentz transformation makes the dipole radiation extremely forward peaked → TR photons stay close to particle track



$\theta << mrad$

- Typical photon energy: $\hbar\omega \approx \frac{1}{4}\hbar\omega_p\gamma$

$e^{\pm}$ at E = 10 GeV; → $\gamma \sim 2 \cdot 10^4$

E_γ ~ 0.25 · 20 eV · $2 \cdot 10^4$ ~ 10 keV → X-rays

- Photons / boundary: $N_{ph} \approx \frac{W}{\hbar\omega} \propto \alpha \approx \frac{1}{137}$

- Energy loss by TR $\left.\frac{dE}{dx}\right|^{TR} = N_{ph} \cdot \mathrm{E}_\gamma = \alpha \cdot 10\ \mathrm{keV} \approx 100\ \mathrm{eV/boundary}$

(TR detectors have many boundaries!)

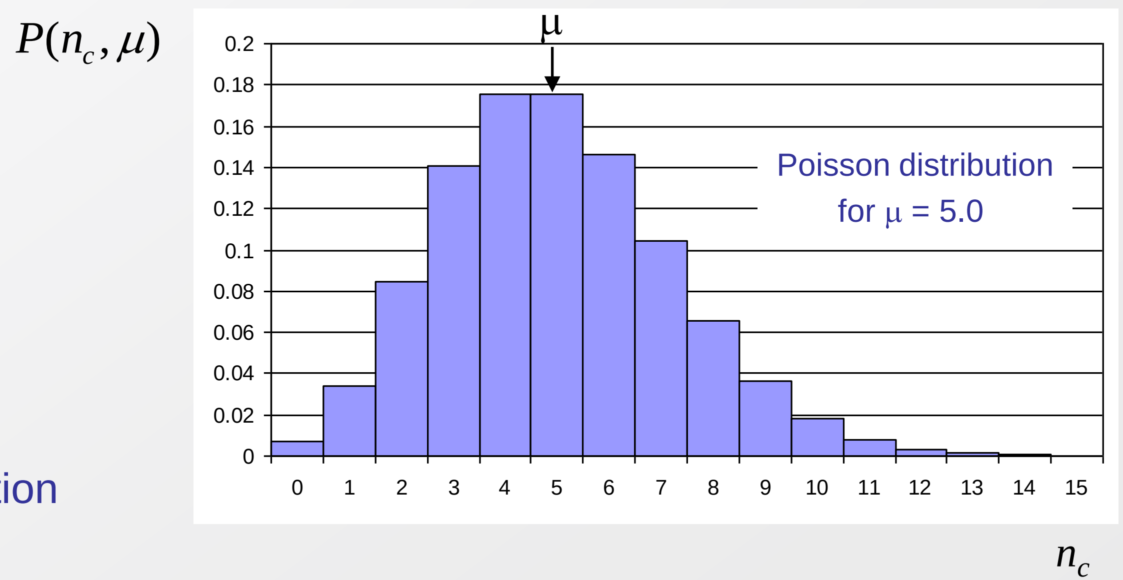
Detector resolution – some general observations

Energy deposition in detectors happens in small discrete and independent steps.

Even in the case of a well defined and constant amount of energy deposited in a detector, the achievable resolution in terms of energy, spatial coordinates or time is constrained by the statistical fluctuations in the number of charge carriers (electron-hole pairs, electron-ion pairs, scintillation photons) produced in the detector.

In most cases, the number of charge carriers n_c is well described by a Poisson distribution with mean $\mu = \langle n_c \rangle$

$$P(n_c, \mu) = \frac{\mu^{n_c} e^{-\mu}}{n_c!}$$



The variance of the Poisson distribution is equal to its mean value.

$$\sigma_{n_c}^2 = \langle n_c^2 \rangle - \langle n_c \rangle = \langle n_c \rangle = \mu$$

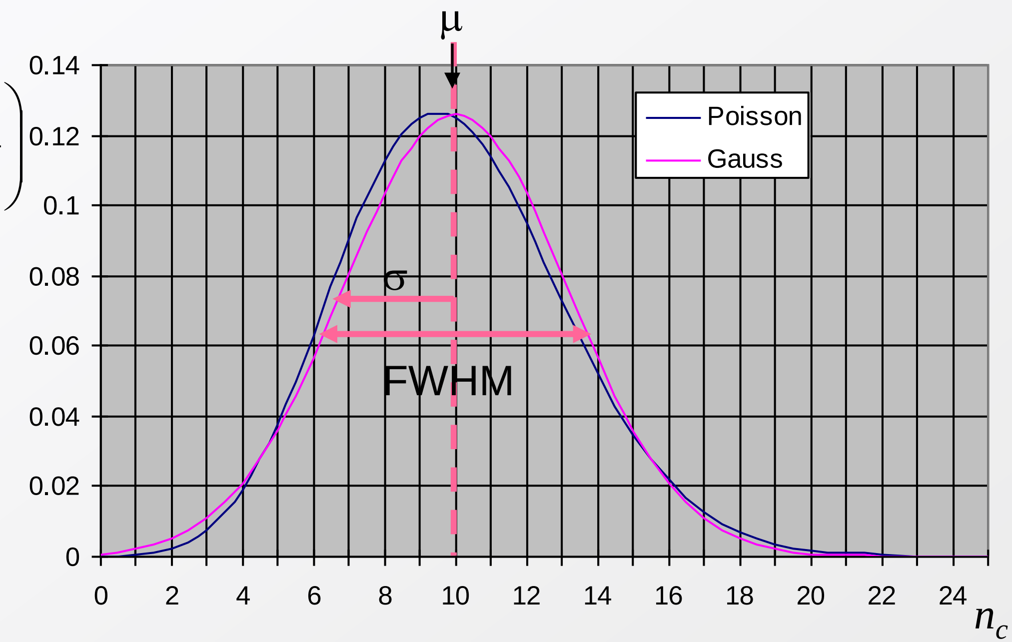
$$\sigma_{n_c} = \sqrt{\mu}$$ standard deviation

Detector resolution

For large μ values, (e.g. μ > 10) the Poisson distribution becomes reasonably well approximated by the symmetric and continuous Gauss distribution.

$$G(n_c,\mu) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(n_c-\mu)^2}{2\sigma^2}\right)$$

$$\text{with } \sigma = \sqrt{\mu}$$



Often used to characterize detector resolution: FWHM

$$\exp\left(-\frac{(n_c-\mu)^2}{2\sigma^2}\right) \equiv \frac{1}{2}$$

$$\rightarrow \quad n_c - \mu = \sqrt{-2\ln 0.5}\cdot\sigma$$

$$\text{FWHM} = 2(n_c-\mu) = 2.35\cdot\sigma$$

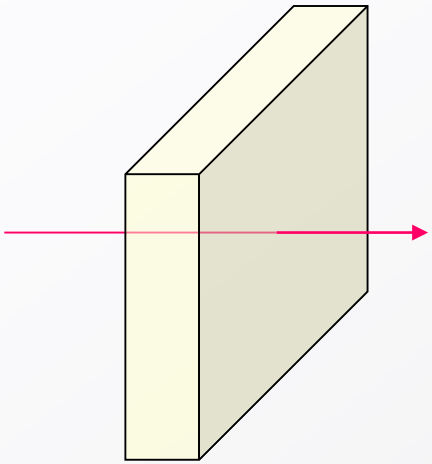
The energy resolution of many detectors is found to scale like

$$\sigma_E \propto \sqrt{\langle n_c\rangle} = \sqrt{\mu} \quad \rightarrow \quad \frac{\sigma_E}{E} \propto \frac{\sqrt{\mu}}{\mu} = \frac{1}{\sqrt{\mu}}$$

Often, also time and spatial resolution improve with increasing $\langle n_c \rangle$

$$\frac{\sigma_{x,t}}{x,t} \propto \frac{1}{\sqrt{\mu}}$$

Case 1: particle traversing the detector



The measured energy deposition E is derived from the number of charge carriers (e.g. e-h or e-ion pairs)

$$\langle n_c \rangle = \mu = a \cdot \frac{dE}{dx} \Delta x \qquad \frac{\sigma_E}{E} \approx \frac{\sqrt{\mu}}{\mu} = \frac{1}{\sqrt{\mu}}$$

(Poisson limit)

! We ignore here many effects which, in a real detector, will degrade the resolution beyond the Poisson limit !

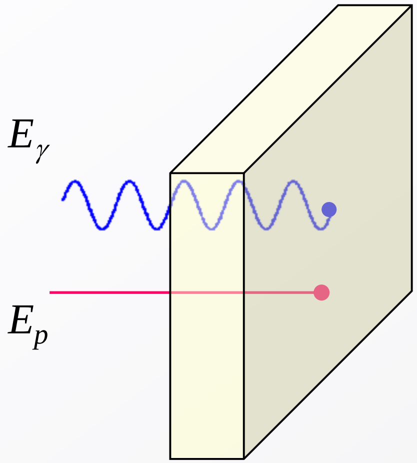
$$\frac{\sigma_E}{E} \geq \frac{1}{\sqrt{\mu}}$$

In particular, depending on the thickness Δx and density, there may also be strong Landau tails.

Energy resolution – Fano factor

Case 2: Particle stops in the detector, or (x-ray) photon is fully absorbed

→ the energy deposition in the detector is fixed (but may vary from event to event).



$$\langle n_c \rangle = \mu = a \cdot E_{p,\gamma} \qquad \frac{\sigma_E}{E} = \frac{\sqrt{F}}{\sqrt{\mu}}$$

F = Fano factor

Ugo Fano,
Phys. Rev. 72 (1947), 26–29

(1912 - 2001)

For a formal derivation: See e.g. book by C. Grupen, pages 15-18.

For a physics motivated derivation: See e.g. H. Spieler, Heidelberg Lecture notes, ch. II, p25…

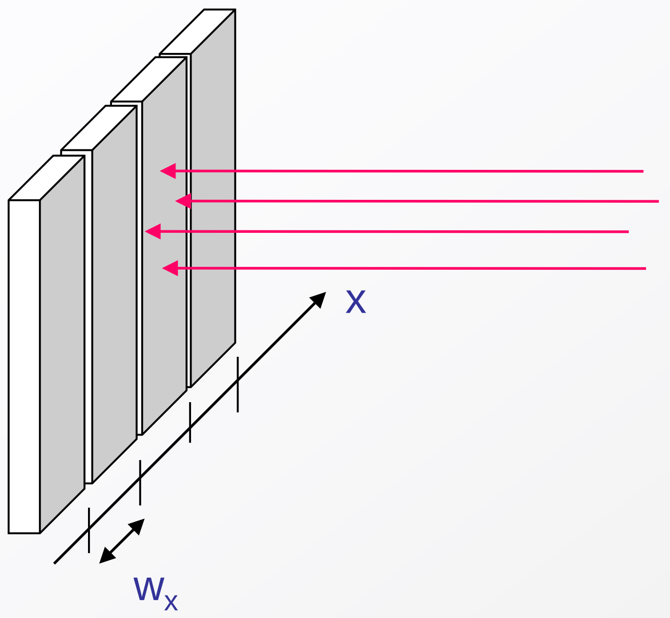
The fluctuation in the number of produced charge carriers n_c is constrained by energy conservation. The discrete steps are no longer fully independent. Their fluctuation can be smaller than the Poisson limit, i.e. $F < 1$.

$F_{\mathrm{Si}} = 0.12,$

$F_{\mathrm{diamond}} = 0.08.$

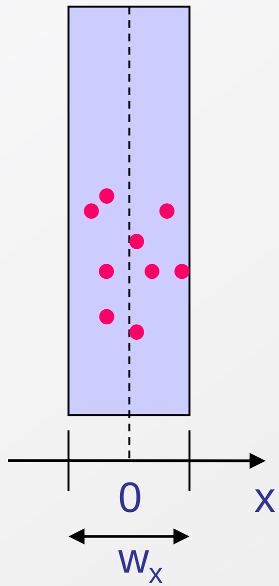
The detector resolution is (can be?) improved by a factor $\sqrt{F}$

Resolution of a discrete detector



Assume a detector consisting of strips of width w_x, exposed to a beam of particles. The detector produces a (binary) signal if one of the strips was hit.

What is its resolution σ_x for the measurement of the x-coordinate?



Consider one strip only (for simplicity at $x = x_d = 0$):

For every recorded hit, we know that the strip at $x = 0$ was hit, i.e. the particle was in the interval

$$-\frac{w_x}{2} \leq x \leq \frac{w_x}{2}$$

$$\Delta x = (x - x_d) = (x - 0) = x$$

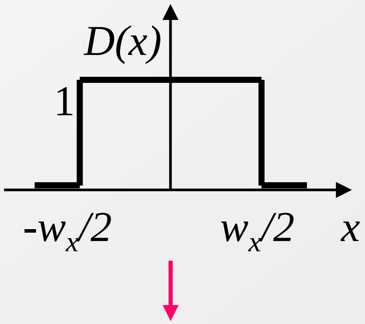
Resolution of a discrete detector (cont.)

$$\sigma_x^2 = \left\langle (x-0)^2 \right\rangle = \frac{\sum_i (x-0)^2}{\sum_i 1} \quad \rightarrow \sigma_x^2 = \frac{\int_{-w_x/2}^{w_x/2} (x-0)^2 \cdot dx}{\int_{-w_x/2}^{w_x/2} dx} = \frac{\frac{1}{3} x^3 \Big|_{-w_x/2}^{w_x/2}}{w_x} = \frac{w_x^2}{12}$$

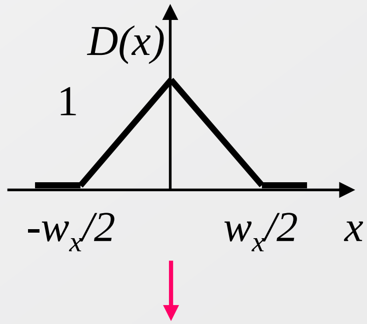
$$\sigma_x = \frac{w_x}{\sqrt{12}}$$

We have tacitly assumed that the particles are uniformly distributed over the strip width. More generally, a distribution function $D(x)$ needs to be taken into account

$$\sigma_x^2 = \frac{\int_{-w_x/2}^{-w_x/2} (x-0)^2 D(x) \cdot dx}{\int_{-w_x/2}^{-w_x/2} D(x) \cdot dx}$$



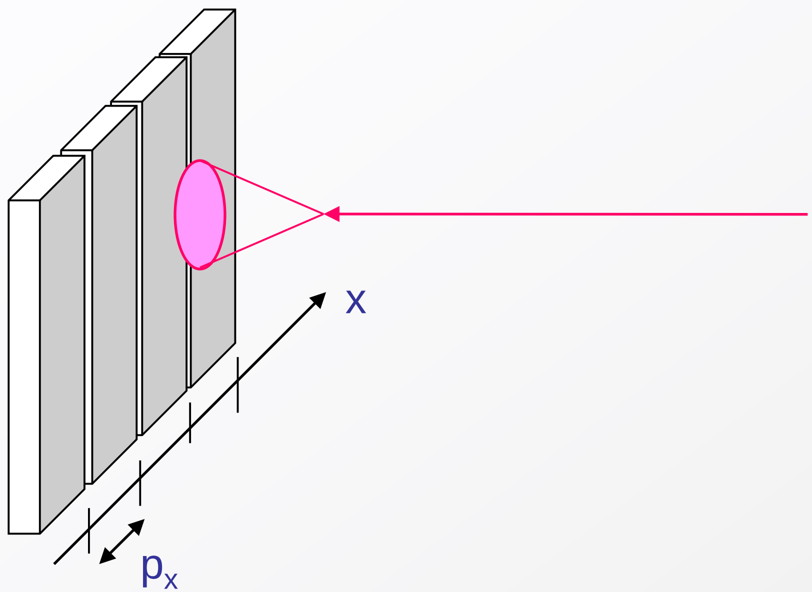
$$\sigma_x = \frac{w_x}{\sqrt{12}}$$



$$\sigma_x = \frac{w_x}{\sqrt{24}}$$

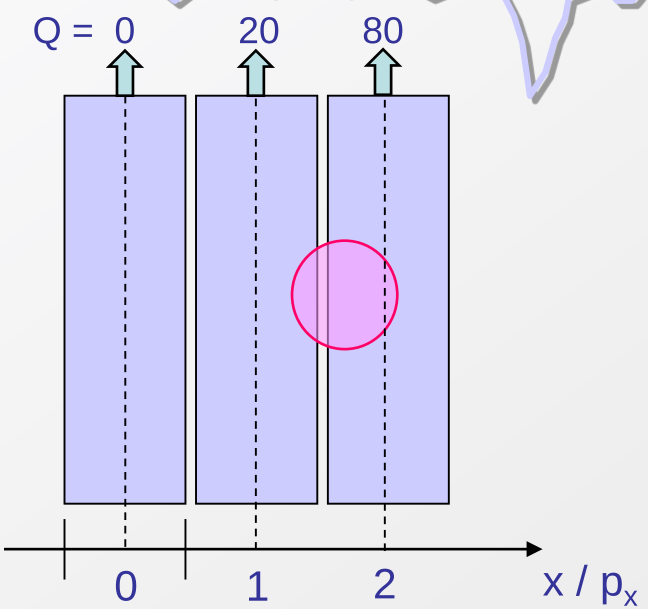
Detector spatial resolution

Resolution of an analog detector



Assume a detector where the charge distribution is larger than its pitch p_x. Every hit cell produces an (analog) signal corresponding to the deposited charge.

What is its resolution σ_x for the measurement of the x-coordinate?



Centroid calculation

$$x = \frac{1}{Q_{tot}} \sum Q_i \cdot x_i$$

$$x = \frac{1}{100}(0 \cdot 0 + 20 \cdot 1 + 80 \cdot 2) = 1.8$$

$$\sigma_x \propto \frac{ENC}{Q_{tot}} p_x$$ (Equivalent noise charge)

Depending on charge and noise, the resolution can be much better than in the discrete case.

Acknowledgements

- Most of my slides are based on material I prepared in the last two decades for other courses (CERN academic training, various schools).
- Some of the courses were given together with colleagues, from whom I ‘borrowed’ plots and figures
- Leszek Ropelewski, Thierry Gys, Carmelo D’Ambrosio, Arnaud Foussat, Rob Veenhof, Michael Hauschild (list is certainly not complete!)
- Many plots and ideas come from the excellent text books listed on the previous slide.
- Lots of good material can nowadays be found by just *Googling*. The authors are then not always recognizable/acknowledgeable.

Back-up